Week in Review # 8

1. \( x \) = the number of senior tickets sold.
   \( y \) = the number of adult tickets sold.
   \( z \) = the number of children tickets sold.
   \[ x + y + z = 700 \]
   \[ 6x + 8y + 3.5z = 3512.5 \]
   \[ 3y = z \]

2. \( x \) = the number of Boeing 747s bought.
   \( y \) = the number of Boeing 777s bought.
   \( z \) = the number of Airbus A321s bought.
   \[ x + y + z = 11400 \]
   \[ 400x + 300y + 200z = 3200 \]
   \[ 200x + 160y + 60z = 1540 \]

3. \( x \) = the amount invested in low-risk stocks.
   \( y \) = the amount invested in high-risk stocks.
   \( z \) = the amount invested in bonds.
   \[ x + y + z = 82000 \]
   \[ y = x + z \]
   \[ 0.08x + 0.15y + 0.04z = 9050 \]

4. (a) no solution
   (b) \( x = 9, \ y = 10, \) and \( z = 6 \)
   (c) \( x = 2 - 4z \)
   \[ y = 9 - 5z \]
   \[ z = \text{any number} \]
   (d) \( x = 7 - 2y - 2w \)
   \[ z = 3 - 4w \]
   \[ y = \text{any number} \]
   \[ w = \text{any number} \]
   (e) \( x = 4, \ y = 2, \) and \( z = 8 \)

5. The row operations that need to be performed are: \( R_2 + 5R_1 \rightarrow R_2 \) and \( R_3 + (-4)R_1 \rightarrow R_3 \)

\[
\begin{bmatrix}
1 & 0 & 9 & | & 12 \\
0 & 2 & 46 & | & 63 \\
0 & 2 & -39 & | & -40 \\
\end{bmatrix}
\]

6. \[
\begin{bmatrix}
3 & 0 & 23 & | & 17 \\
7 & 11 & 39 & | & 25 \\
10 & 0 & 1 & | & 16 \\
0 & 5 & 6 & | & 1 \\
\end{bmatrix}
\]

7. (a) first rewrite the equations as shown.
   \[ 3x + y = 9 \]
   \[ x - y + z = 4 \]
   \[ 3x + z = 11 \]
   \[ 4x - y + 2z = 15 \]

\[
\begin{bmatrix}
3 & 1 & 0 & 9 \\
1 & -1 & 1 & 4 \\
3 & 0 & 1 & 11 \\
4 & -1 & 2 & 15 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Answer: \( x = 2, \ y = 3, \) and \( z = 5 \)

(b) \[
\begin{bmatrix}
1 & 3 & 1 & 10 \\
2 & 7 & -1 & 21 \\
4 & 13 & 1 & 41 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 10 & 7 \\
0 & 1 & -3 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Answer: \( x = 7 - 10z \)
\[ y = 1 + 3z \]
\[ z = \text{any number} \]

Note: no restrictions can be placed on the parameter since this was not a word problem.

(c) \[
\begin{bmatrix}
3 & 2 & 5 & 7 \\
1 & 4 & 1 & 13 \\
4 & -5 & 2 & -9 \\
5 & 10 & 7 & 32 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

Answer: no solution.

8. (a) Set up of the problem:
   \( x \) = the number of old dvds bought
   \( y \) = the number of new dvds bought
   \( z \) = the number of semi-new dvds bought

\[ x + y + z = 60 \]
\[ 10x + 16y + 22z = 840 \]

Solution:
\[ x = 20 + z \]
\[ y = 40 - 2z \]
\[ z = \text{any number} \]

Now place restrictions on the parameter \( z \). This is the mathematical process. You could also do this by inspecting the parametric solution for what values of \( z \) will make sense.

We know that the number of dvds bought must be greater than or equal to zero.
\[ x \geq 0 \quad y \geq 0 \quad z \geq 0 \]
\[ 20 + z \geq 0 \quad 40 - 2z \geq 0 \]
\[ z \geq -20 \quad 40 \geq 2z \]
\[ 20 \geq z \]

We also know that the number of dvds bought must be less than 60.
\[ x \leq 60 \quad y \leq 60 \quad z \leq 60 \]
\[ 20 + z \leq 60 \quad 40 - 2z \leq 60 \]
\[ z \leq 40 \quad -2z \leq 20 \]
\[ z \geq -10 \]

Thus we get that \( 0 \leq z \leq 20 \) and \( z \) must be an integer or in other words \( z = 0, 1, 2, 3, \ldots, 20 \)

(b) 21 different solutions.

9. (a) \[3d_{2,2} + 2c_{2,1} = 3(5) + 2(-2) = 11\]

(b) \[
\begin{bmatrix}
21 & 6 & 12 \\
18 & 15 & 0
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & -2 & 2 \\
3 & 5 & 0
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
25 & 8 & 4 \\
4 & 9 & 16
\end{bmatrix}
\]

(e) \[
\begin{bmatrix}
-15 & -6 & 12 \\
22 & 7 & -32
\end{bmatrix}
\]

(f) not possible, wrong sizes.

(g) \[
\begin{bmatrix}
-9 & 3 \\
14 & 15 \\
2 & 0
\end{bmatrix}
\]

10. simplify the left and right side.
\[
\begin{bmatrix}
19 & 8x-3y \\
4y-18 & 10
\end{bmatrix} = \begin{bmatrix}
19 & -28 \\
x & 10
\end{bmatrix}
\]

Now solve
\[8x - 3y = -28\]
\[4y - 18 = x\]

Answer: \( x = -2, \ y = 4 \)