Exam # 2 Sample Review
Sections 2.1-2.4, 3.1-3.5, and 4.1-4.3

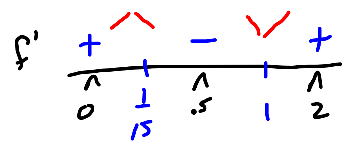
1. Let $f(x) = 5x^3 - 8x^2 + x + 70$

   (a) Find all critical values of $f(x)$ and classify them as local max, local min, or neither.

$f'(x) = 15x^2 - 16x + 1 = (15x-1)(x-1)$ domain is all Real #s.

$0 = (15x-1)(x-1)$

C.V. $x = \frac{1}{15}$ $x = 1$



Local max at $x = \frac{1}{15}$

Local min at $x = 1$

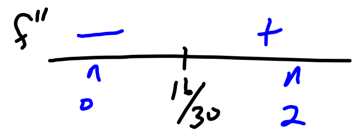
   (b) Find the inflection values.

$f'' = 30x - 16$

$0 = 30x - 16$

$16 = 30x$

$x = 16/30$



Inflection point at $x = 16/30$

   (c) Find the interval(s) where $f(x)$ is decreasing and concave down.

$\left(\frac{1}{15}, \frac{16}{30}\right)$

   (d) Find the interval(s) where $f(x)$ is increasing and concave up. $(1, \infty)$

   (e) Find the global maximum and global minimum of $f(x)$ on the interval $[0, 6]$

C.V. $x = \frac{1}{15}$ $x = 1$

$f(x)$ is cont. on the closed interval $[0,6]$

$f\left(\frac{1}{15}\right) = 70.033$

$f(1) = 68$

$f(0) = 70$

$f(6) = 868$

global max = 868

global min = 68

2. Let $f(x) = 2x^4 - 10x^3 + 10$.

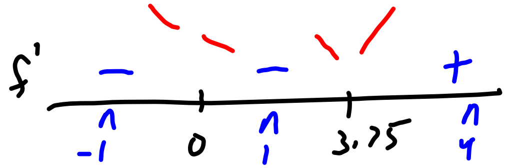
(a) Find the intervals where $f(x)$ is increasing and where it is decreasing. Classify the critical values.

$f'(x) = 8x^3 - 30x^2$

$= x^2 \cdot (8x - 30)$

$0 = x^2 \qquad 0 = 8x - 30$

$x = 0 \qquad x = 30/8 = 3.75$



Inc $(3.75, \infty)$

dec $(-\infty, 0)\ (0, 3.75)$

Local min at $x = 3.75$

$x = 0$ is a neither.

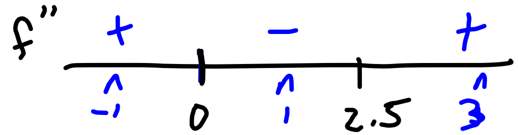
(b) Find the intervals where $f(x)$ is concave up and where it is concave down. Find the x-values of the inflection points.

$f''(x) = 24x^2 - 60x$

$= 12x(2x - 5)$

$0 = 12x \qquad 0 = 2x - 5$

$x = 0 \qquad x = \frac{5}{2} = 2.5$



C.U. $(-\infty, 0)$

$(2.5, \infty)$

c.d $(0, 2.5)$

Inflection point

at $x = 0$ & $x = 2.5$

2. Let $f(x) = 2x^4 - 10x^3 + 10$.

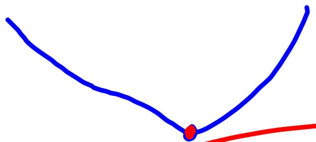
even poly. that opens up so no global max,

(c) Does the function have a global max? If so, where?

(d) Does the function have a global min? If so, where?

by this: <u>yes</u>

Look at part A for shape.



at $x=3.75$

(e) Does the function have a global max on the $[1, 4]$? If so, where?

cont. function & closed interval so yes.

C.V. $x=0$

$x=3.75$

not in the interval.

$f(1) = 2$

$f(4) = -118$

$f(3.75)$ ← by part d global min.

global max at $x=1$

3. Find the x-values of the inflection points for $f(x)$ if the domain of $f(x)$ is all real numbers and $f''(x) = 3x^2(x+6)(7-x)^3$

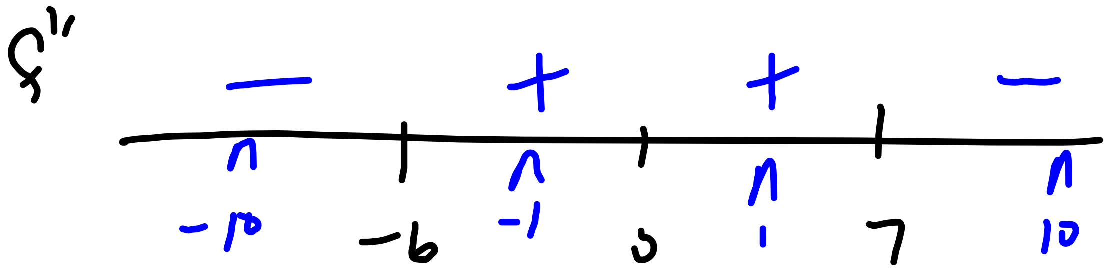
$0 = f''(x)$

$(7-x)^3 = 0$

$7 - x = 0$

$7 = x$

$x = 0 \quad x = -6 \quad x = 7$



Inflection point at $x = -6$ & $x = 7$

4. Find the constants a and b such that the point $(1, -4)$ is an inflection point for $f(x) = ax^3 + bx^2 + 10$.

$f'(x) = 3ax^2 + 2bx$

$f''(x) = 6ax + 2b$

$f''(1) = 0$

$6a + 2b = 0$

$2b = -6a$

$b = -3a$

$f(1) = -4$

$a + b + 10 = -4$

$a + b = -14$

$a - 3a = -14$

$-2a = -14$

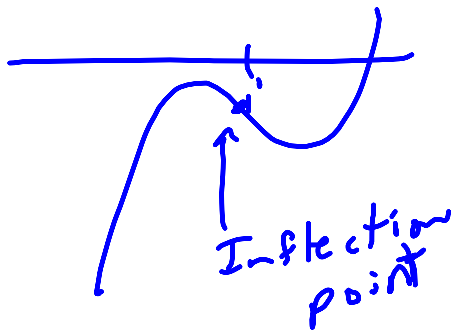
$a = 7$

$b = -21$

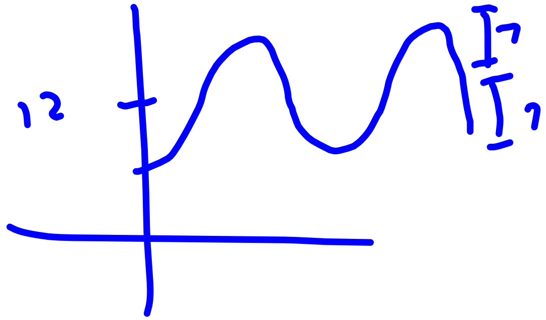
$y = 7x^3 - 21x^2 + 10$

5. For problem 4, is the critical value that is less than 1 a local max or a local min? Answer this question without performing any calculations or using the graphing calculator.

Local max



6. Find the global max and the global minimum for $y = 12 - 7\cos(3x)$



Amp = 7

middle is 12

global max = $12 + 7 = 19$

global min = $12 - 7 = 5$

7. Give the formula for a function that has a global maximum but doesn't have a global minimum.

$y = -x^2$ or $y = -32x^{40} + 3x + 1$

8. Let $f''(x) = 6x - 15$. What can be concluded about the following values of x.

(a) $x = 3$

$f''(3) = 18-15 = 3$

Concave up.

(b) $x = 5$ if $\underline{f'(5) = 0}$

$f''(5) = 30-15 > 0$

$\underline{x=5 \text{ is a c.v.}}$

$\underline{\text{concave up.}}$

Local min at $x=5$

(c) $x = 0$ if $\underline{f'(0) = 0}$

$\underline{f''(0) = -15}$

c.v.

concave down

local max at $x=0$

9. Sketch a graph of a function that has these properties.

$f(x)$ is continuous for all real numbers.

$f(x) < 5$ for all real numbers.

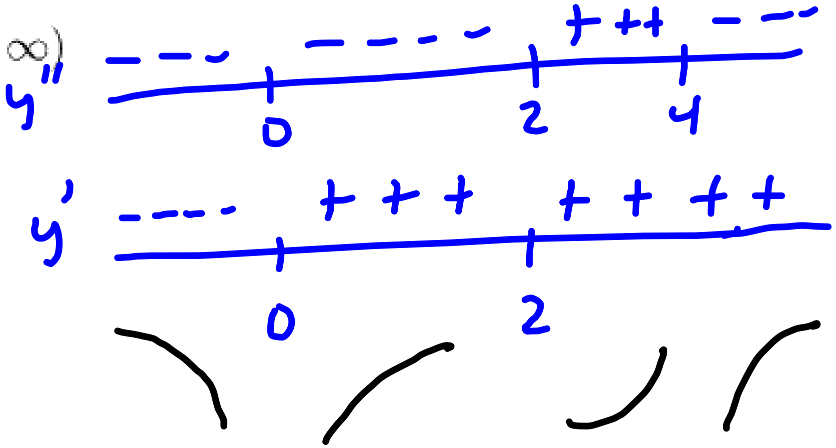
$f'(2) = 0$ and $f(2) = 1$

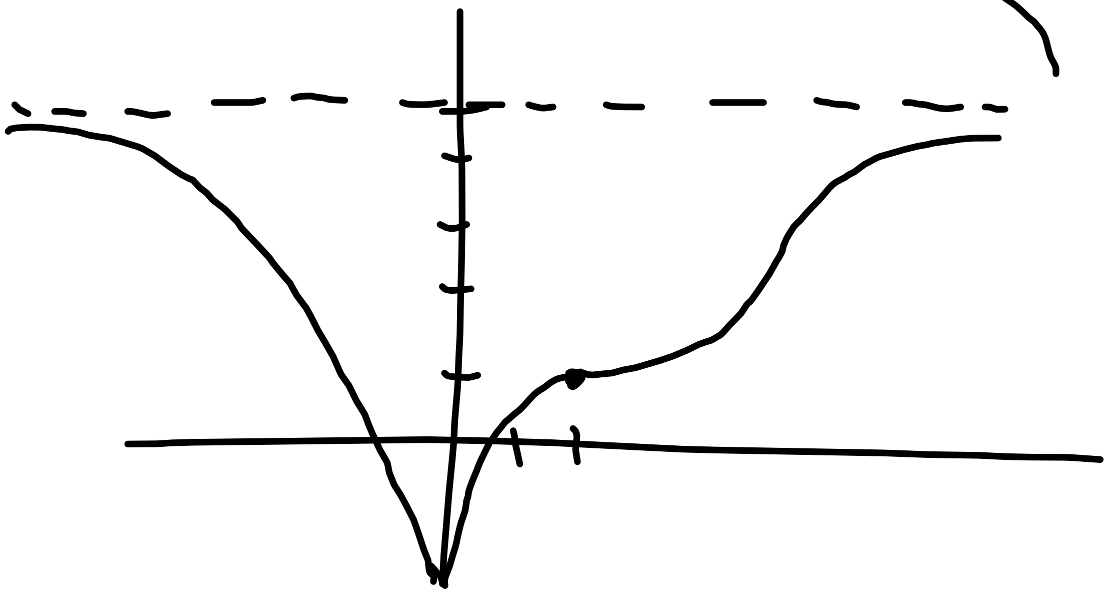
$f'(x) < 0$ on $(-\infty, 0)$

$f'(x) > 0$ on $(0, 2)$ and $(2, \infty)$

$f''(x) < 0$ on $(-\infty, 0)$ and $(0, 2)$ and $(4, \infty)$

$f''(x) > 0$ on $(2, 4)$





10. The value of a van purchased in 2005 can be approximated by the function $V(x) = 25(0.85)^x$, where x is time in years from 2005, and V is the value in thousand of dollars. Evaluate $V'(4)$ and interpret the result.

$V'(x) = 25(.85)^x \cdot \ln(.85)$

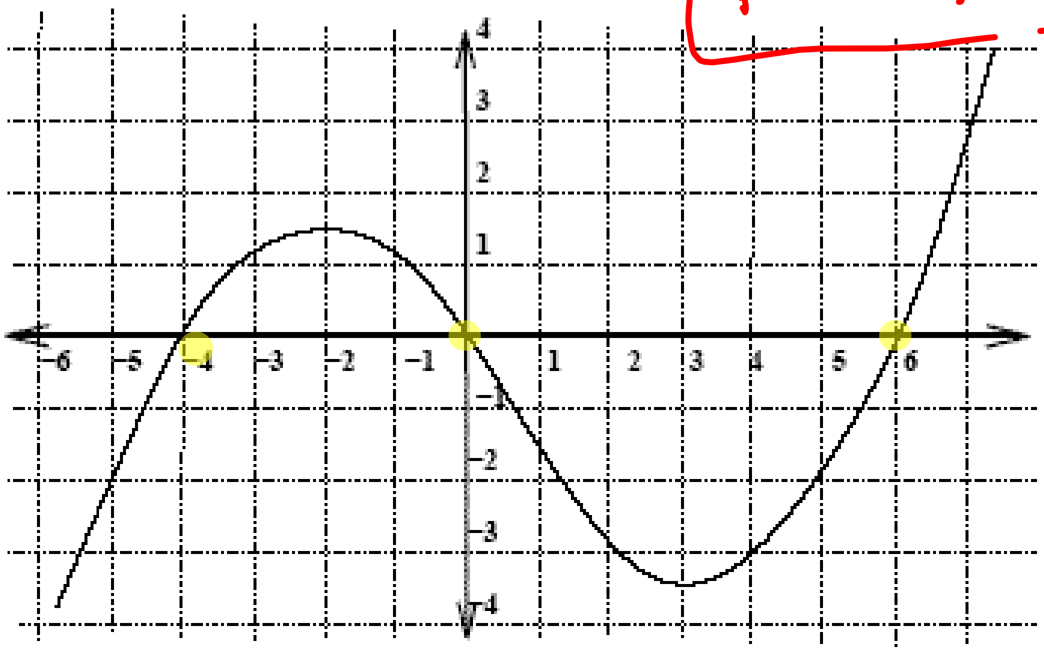
$V'(4) = -2.120897$ Thousand of \$ per yr

↳ \$ -2120.90/yr

In 2009 the value of the van will decrease by approximately \$2120.90 in the next yr.

11. Use the graph to answer these questions.

$f'' > 0 \Leftrightarrow f'$ inc $\Leftrightarrow f$ c.u.
$f'' < 0 \Leftrightarrow f'$ dec $\Leftrightarrow f$ c.d



(a) If the graph is of $f'(x)$, where is $f(x)$ increasing and concave up?

$f'(x)$ pos. $f'(x)$ inc.

$(-4, -2)$, $(6, \infty)$

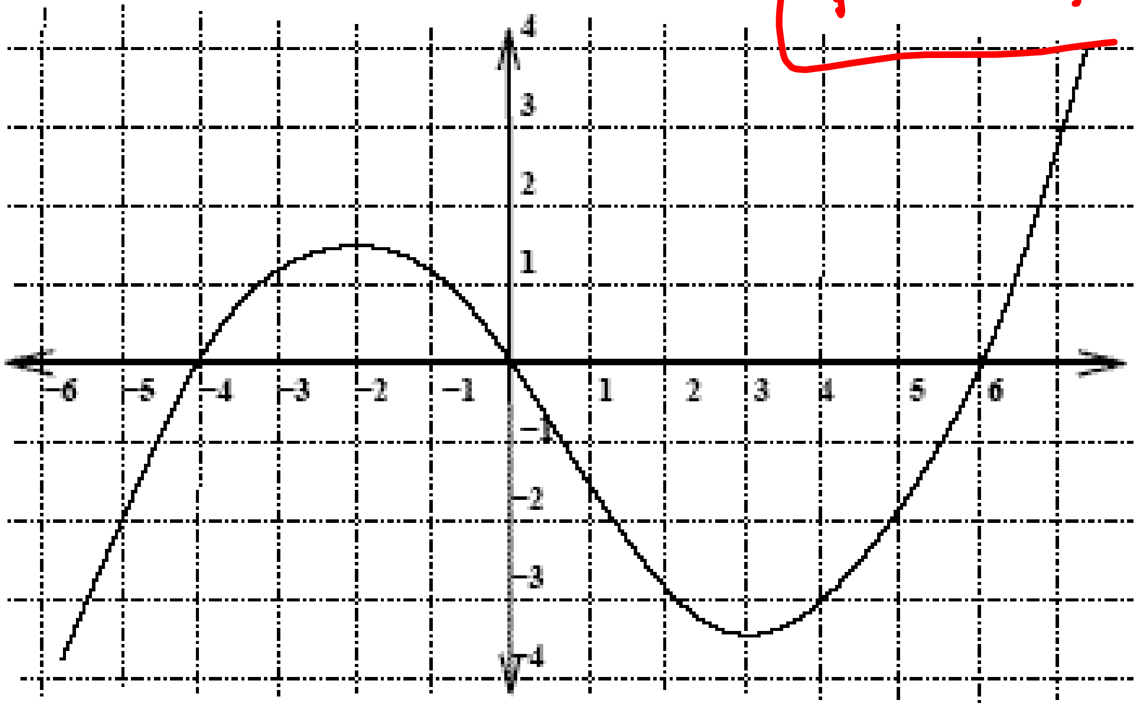
(b) If the graph is of $f'(x)$, where does $f(x)$ have a local minimum?

c.v. at $x = -4, 0, -6$

y' $-$ $+$ $-$ $+$; -4, 0, 6

Local min $x = -4$, $x = 6$

$f'' > 0 \Leftrightarrow f'$ inc $\Leftrightarrow f$ c.u.
$f'' < 0 \Leftrightarrow f'$ dec $\Leftrightarrow f$ c.d



(c) If the graph is of $f'(x)$, where does $f(x)$ have inflection points?

↳ where $f'(x)$ changes direction

$x = -2$

$x = 3$

(d) If the graph is of $f''(x)$, where is $f(x)$ concave up? $(-4, 0), (6, \infty)$

12. Find the equation of the tangent line $y = \ln(x^3 - 7) + e^{x^2-4}$ at $x = 2$

$f(2) = \ln(8-7) + e^{4-4} = \ln(1) + e^0 = 0 + 1 = 1$

point $(2, 1)$

$f'(x) = \frac{3x^2}{x^3-7} + 2x\,e^{x^2-4}$

$m_{tan} = f'(2) = \frac{12}{8-7} + 4e^{4-4} = 12 + 4 = 16$

$\boxed{y - 1 = 16(x-2)}$

13. Find $f''(x)$ for $f(x) = e^{x^2+4x}$

$$f' = \underbrace{(2x+4)}_{f}\underbrace{e^{x^2+4x}}_{g}$$

$$f'' = \underbrace{2}_{f'}\underbrace{e^{x^2+4x}}_{g} \underset{+}{+} \underbrace{(2x+4)}_{f}\cdot\underbrace{(2x+4)e^{x^2+4x}}_{g'}$$

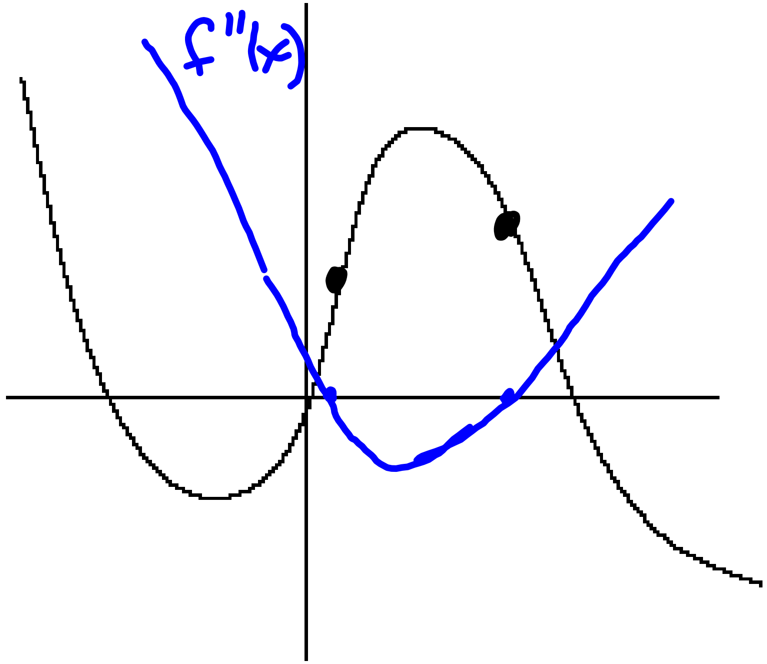
If you simplify.

$$f'' = 2e^{x^2+4x} + (4x^2+16x+16)e^{x^2+4x}$$

$$= [2 + 4x^2+16x+16]e^{x^2+4x}$$

$$= (4x^2+16x+18)e^{x^2+4x}$$

14. Sketch the graph of the 2nd derivative for this function.

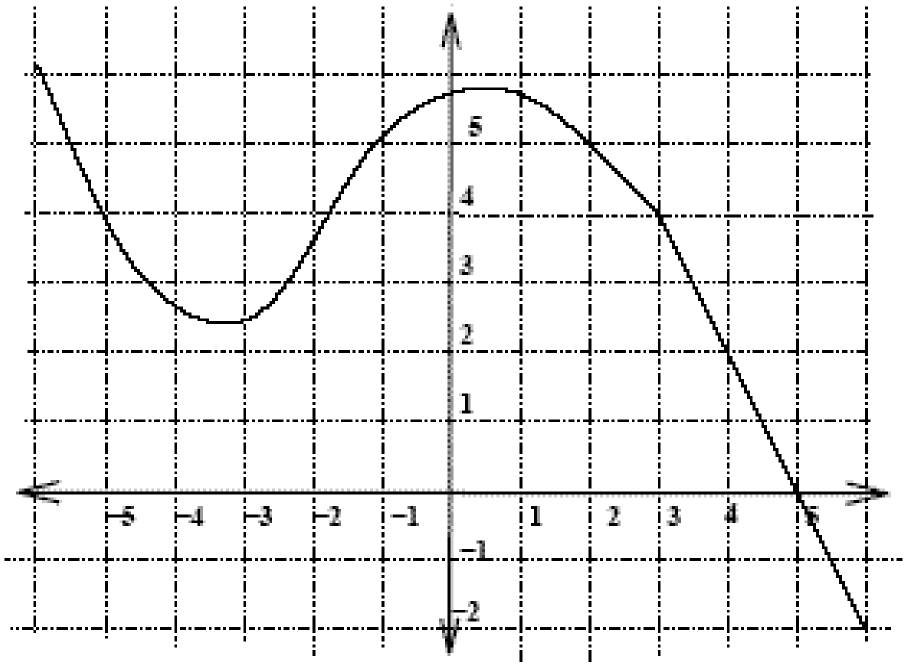


Inflection points on f(x) are black dots.

15. Let $f(x)$ and $g(x)$ whose values and derivative values are given by the table. Let $k(x)$ be the function given in the graph. Use this information to compute the following.

x	0	1	2
$f(x)$	1	-1	4
$f'(x)$	2	4	6
$g(x)$	-1	2	3
$g'(x)$	5	3	5

k(x)



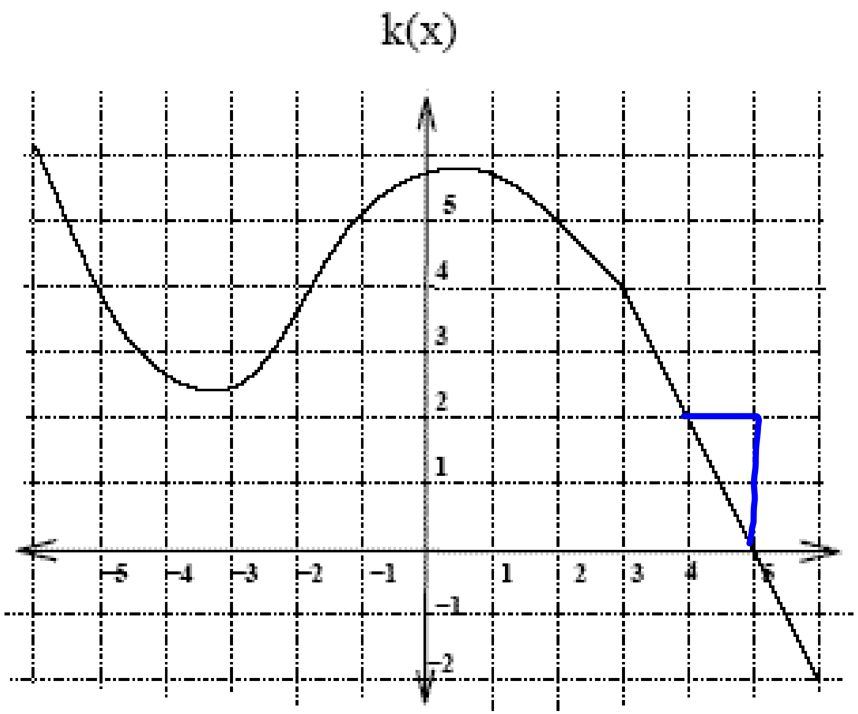
(a) If $M(x) = \cos(f(x))$, find $M'(0)$

$m'(x) = -\sin(f(x)) \cdot f'(x)$

$m'(0) = -\sin(f(0)) \cdot f'(0)$

$= -\sin(1) \cdot 2 \quad = -2\sin(1)$

x	0	1	2
$f(x)$	1	-1	4
$f'(x)$	2	4	6
$g(x)$	-1	2	3
$g'(x)$	5	3	5



(b) If $H(x) = x^2 - (k(x))^3$, find $H'(4)$

$H'(x) = 2x - 3(k(x))^2 \cdot k'(x)$

$H'(4) = 8 - 3(k(4))^2 \cdot k'(4)$

$= 8 - 3(2)^2 \cdot -2$

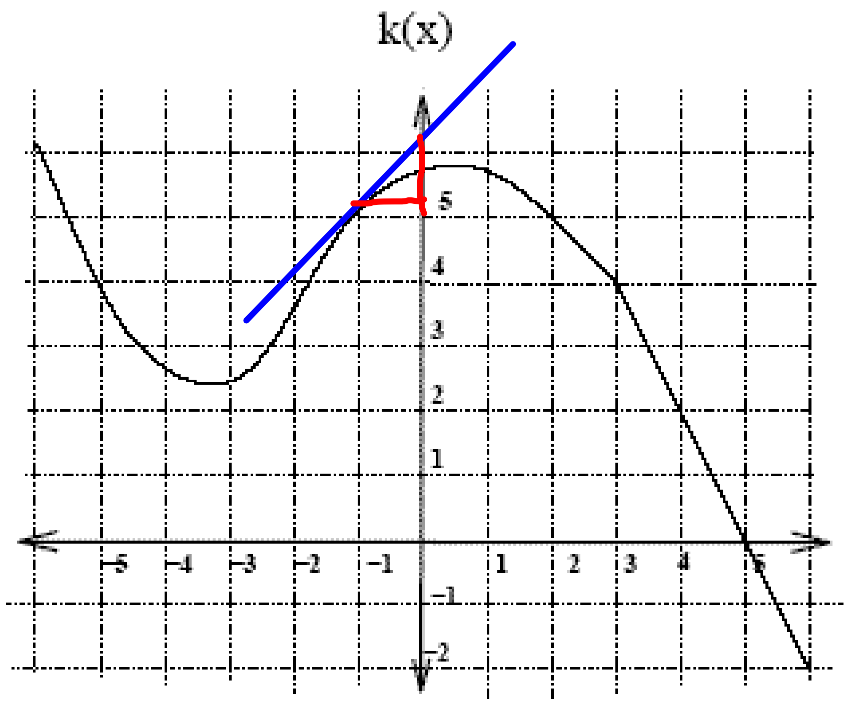
$= 8 - 3(4)(-2)$

$= 8 + 24 = 32$

$k(4) = 2$

$k'(4) = \frac{-2}{1}$

x	0	1	2
$f(x)$	1	-1	4
$f'(x)$	2	4	6
$g(x)$	-1	2	3
$g'(x)$	5	3	5



(c) If $J(x) = k(x)g(x^2)$, find $J'(-1)$

$$J'(x) = k'(x)\, g(x^2) + k(x) \cdot g'(x^2) \cdot 2x$$

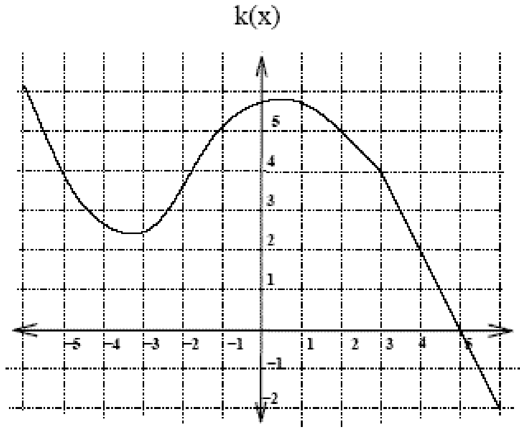
$$J'(-1) = k'(-1)\, g((-1)^2) + k(-1) \cdot g'((-1)^2) \cdot 2(-1)$$

$$= k'(-1)\, g(1) + k(-1) \cdot g'(1)\, (-2)$$

$$= (1)(2) + (5.1)(3)(-2)$$

$$= 2 + (5.1)(-6) = \underline{\qquad}$$

x	0	1	2
$f(x)$	1	-1	4
$f'(x)$	2	4	6
$g(x)$	-1	2	3
$g'(x)$	5	3	5



(d) If $N(x) = \dfrac{f(x) + 2x}{g(x)}$, find $N'(2)$

$$N'(x) = \frac{g(x)\left(f'(x)+2\right) - \left(f(x)+2x\right)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$N'(2) = \frac{g(2)\left(f'(2)+2\right) - \left(f(2)+4\right)\cdot g'(2)}{\left(g(2)\right)^2}$$

$$= \frac{3\,(6+2) - (4+4)(5)}{(3)^2}$$

$$= \frac{3\,(8) - 8(5)}{9} = \frac{24-40}{9}$$

$$= \frac{-16}{9}$$