1. A bat starts out traveling towards the exit of a tunnel 90 feet away. The graph describes the bat’s velocity, $v(x)$ in ft/sec, vs. time.

(a) What is the interpretation of $\int_{1}^{4} v(x)dx$? give the units of the definite integral.

Since $v(x) > 0$ for $1 < x < 4$, $\int_{1}^{4} v(x)dx$ is the distance traveled.

$\int_{4}^{7} v(x)dx$ is the total change in the bat’s position from the 4 sec mark to the 7 sec mark.

(b) When does the bat change direction?

at $x = 5$, $x = 7$, $x = 10$
(c) How far from the starting point is the bat after 6 seconds?

\[
\text{\underline{70 ft from start.}}
\]

(d) How far has the bat traveled in the 6 seconds?

\[
\text{\underline{90 ft.}}
\]

(e) Does the bat make it out of the tunnel in 10 seconds?

\[
80 - 20 + 50 = 110 \text{ ft}
\]

\[
\text{So yes.}
\]
2. The following is the graph of $f'(x)$. Use the fact that $f(0) = 40$ along with the given areas for the regions to answer the following. Region A = 8, Region B = 48, and Region C = 34.

(a) Find the coordinates, $(x,y)$, for all of the critical values. Also classify the critical values.

Critical Values (C.V.)

- $x = 0$ is a **local min** $(0, 40)$
- $x = 3$ is a **local max** $(3, 88)$
- $x = 7$ is a **neither** $(7, 54)$

$$\int_{0}^{3} f'(x) \, dx = f(3) - f(0)$$

$$40 = f(3) - 40$$

$$88 = f(3)$$

$$\int_{3}^{7} f'(x) \, dx = f(7) - f(3)$$

$$-34 = f(7) - 88$$

$$54 = f(7)$$

(b) Find the x-values for the inflection points.

(c) Sketch the graph of $f(x)$. 

**Note:** The graph shows a curve with marked sections and critical points, but the specific values and calculations are provided in the text.
2. The following is the graph of $f'(x)$. Use the fact that $f(0) = 40$ along with the given areas for the regions to answer the following. Region A = 8, Region B = 48, and Region C = 34.

(b) Find the $x$-values for the inflection points.

\[ x \approx 1.5 \]
\[ x \approx 4.6 \]

(c) Sketch the graph of $f(x)$.

\[
\int_{g'} = f(a) - f(b) \\
\int_{-g} = f(a) - f(b) \\
f(c) = 40 + 8 \\
f(-1) = 46
\]
3. If \( x \) is the number of years from 1990, then the population growth of a city, in millions per year, can be modeled by the formula \( 1.5e^{0.25x} \).

(a) Write an expression that will give the growth of the city \( A \) years after 1990.

\[
\int_{0}^{A} 1.5e^{0.25x} \, dx = 1.5 \left( \frac{1}{0.25} \right) e^{0.25x} \bigg|_{0}^{A} = 6e^{0.25A} - 6e^{0}\]

(b) If the city had a population of 5 million in 1990, find a formula that will give the population \( A \) years after 1990.

\[
\int_{0}^{A} p'(x) \, dx = P(A) - P(0)
\]

\[
6e^{0.25A} - 6 = P(A) - 5
\]
4. If \( f'(x) = 5 \cos(3x^2) \) and \( f(-1) = 3 \) then find \( f(1) \).

\( \text{u-sub does not work on this function.} \)

\[
\int_{-1}^{1} 5 \cos(3x^2) \, dx = f(1) - f(-1)
\]

\( \text{Used } \int_{-1}^{1} \)

\[4.05955 = f(1) - 2\]

\[6.05955 = f(1)\]

5. Compute \( \int_{-1}^{1} \frac{9}{x^2 + \cos(3x)} \, dx = \int_{-1}^{1} 2x^{-2} + \cos(3x) \, dx \)

\[= \left. \frac{2}{x^{-1}} + \frac{1}{3} \sin(3x) \right|_{-1}^{5} \]

\[= -\frac{2}{x} + \frac{1}{3} \sin(3x) \bigg|_{-1}^{5} \]

\[= \left( -\frac{2}{5} + \frac{1}{3} \sin(15) \right) - \left( -\frac{2}{5} + \frac{1}{3} \sin(3) \right) \]
6. (a) Compute \( \int_0^A \frac{x}{1 + x^2} \, dx \):

\[
\begin{align*}
&= \int_0^A \frac{1}{2} \frac{1}{u} \, du \\
&= \frac{1}{2} \left| \ln(u) \right| \bigg|_0^A \\
&= \frac{1}{2} \ln(1 + A^2) - \frac{1}{2} \ln(1) \\
&= \frac{1}{2} \ln(1 + A^2) \\
&= \frac{1}{2} \ln(1 + A^2)
\end{align*}
\]

(b) Does \( \int_0^\infty \frac{x}{1 + x^2} \, dx \) converge or diverge?

Since \( \frac{1}{2} \ln(1 + A^2) \to \infty \) as \( A \to \infty \),

\[\int_0^\infty \frac{x}{1 + x^2} \, dx \text{ diverges.}\]
\[
\int_{1}^{\infty} \frac{1}{x^2} \, dx
\]

\[
\int_{1}^{A} x^{-2} \, dx = \frac{x^{-1}}{-1} \bigg|_{1}^{A} = -\frac{1}{x} \bigg|_{1}^{A}
\]

\[
= -\frac{1}{A} - \left( -\frac{1}{1} \right) = -\frac{1}{A} + 1
\]

as \( A \to \infty \) (gets very large)

\[
-\frac{1}{A} \to 0
\]

Means. \( \int_{1}^{\infty} \frac{1}{x^2} \, dx = 1 \)

Say integral converges.
7. Set up the integral(s) that represent this shaded area.

\[
\int_{A}^{C} h(x) - f(x) \, dx + \int_{C}^{E} h(x) - g(x) \, dx
\]
8. Find the value of $B$ so that the area between the curves $f(x) = 2e^x$ and $g(x) = 3x^2$ from $x = 0$ to $x = B$ will be 110.

\[
\int_{0}^{B} (2e^x - 3x^2) \, dx = 110
\]

\[
\left. (e^x - x^3) \right|_{0}^{B} = (2e^B - B^3) - (2e^0 - 0)
\]

\[
= 2e^B - B^3 - 2
\]

\[
2e^B - B^3 - 2 = 110
\]

\[
e^B - B^3 = 112
\]

\[
\text{only way to solve is by graphing on the calc.}
\]

\[
B = 4.6731
\]
9. Compute these integrals

\[ \int \frac{1}{2x^3} - \sin(6x) + \sqrt[3]{x} \, dx = \int \frac{1}{2} x^{-3} - \sin(6x) + x^{\frac{1}{3}} \, dx \]

\[ = \frac{1}{2} \frac{x^{-2}}{-2} + \frac{1}{6} \cos(6x) + \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C \]

\[ = -\frac{1}{4}x^{-2} + \frac{1}{6} \cos(6x) + \frac{3}{4} x^{\frac{4}{3}} + C \]
(b) \( \int \frac{e^{x-1}}{x^2} \, dx \)

\[
\begin{align*}
\text{Let } u &= x^{-1} \\
\frac{du}{dx} &= -1 \cdot x^{-2} \\
du &= -\frac{1}{x^2} \, dx
\end{align*}
\]

\[
\int -e^u \, du = -e^u + C
\]

\[
\Rightarrow \quad -e^{x^{-1}} + C
\]
(c) \[ \int \frac{5e^{5x} - 8e^{-8x}}{(e^{5x} + e^{-8x})^3} \, dx = \int \frac{1}{u^3} \, du = \int u^{-\frac{3}{2}} \, du \]

\[ u = e^{5x} + e^{-8x} \]

\[ du = (5e^{5x} - 8e^{-8x}) \, dx \]

\[ = \frac{u^{-\frac{1}{2}}}{-\frac{1}{2}} + C \]

\[ = -\frac{1}{2} (e^{5x} + e^{-8x})^{-2} + C \]

or

\[ -\frac{1}{2} \left( \frac{1}{(e^{5x} + e^{-8x})^2} \right) + C \]
10. Compute the area between the x-axis and $y = e^x - 1$ from $x = -2$ to $x = 2$.

\[ \int_{-2}^{2} e^x - 1 \, dx = \text{Int} (e^x - 1, x, -2, 2) = 3.25372 \]

\textbf{This is not the answer.}

\[ \int_{-2}^{2} e^x - 1 \, dx \] is the TOTAL Change in the Area.

You need to check the graph first.

\[ \int_{-2}^{2} e^x - 1 \, dx = \left| \left. - 1.35335 \right| + 4.38905 \right| = 5.524391 \]
11. (a) Estimate $\int_{1}^{9} \frac{1}{x} \, dx$ using a left-hand sum with 4 rectangles.

\[ f(x) = \frac{1}{x} \]

\[
2 \left( \frac{1}{1} \right) + 2 \left( \frac{1}{3} \right) + 2 \left( \frac{1}{5} \right) + 2 \left( \frac{1}{7} \right) \\
= 2 + \frac{2}{3} + \frac{2}{5} + \frac{2}{7} \\
= 3.35238 \\
\]

(b) Is the estimate an over estimate or an under estimate? Justify your answer with a graph.

Over since Rect. go above graph of $\frac{1}{x}$
12. Estimate \( \int_{1}^{10} f(x) \, dx \) using a right sum and the information in the table.

\[
\begin{array}{c|cccc}
  x & 1 & 4 & 8 & 10 \\
 f(x) & 5 & 20 & 16 & 8 & 3 \\
\end{array}
\]

\[
3 \cdot (20) + 4 \cdot (16) + 2 \cdot (8)
\]

if you need graph the points