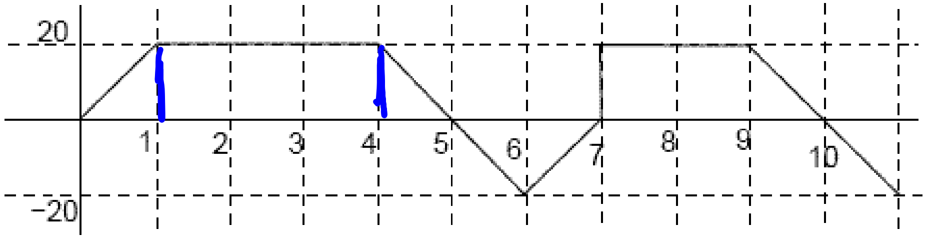
Exam # 3 Sample Review
Sections 5.1-5.5, and 7.1-7.4

1. A bat starts out traveling towards the exit of a tunnel 90 feet away. The graph describes the bat's velocity, $v(x)$ in ft/sec, vs. time.



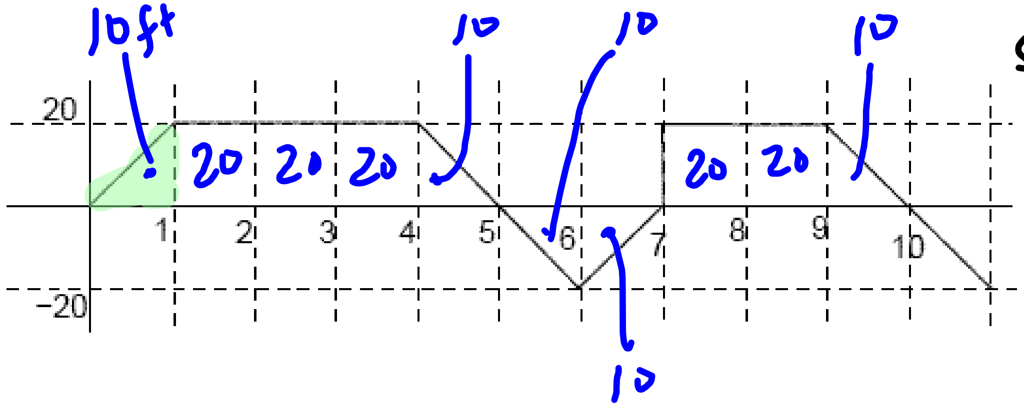
(a) What is the interpretation of $\int_1^4 v(x)dx$? give the units of the definit integral.

since $v(x) > 0$ for $x=1$ to $x=4$ $\int_1^4 v(x)\,dx$ is the distance traveled.

$\int_4^7 v(x)\,dx$ is the total change in the bat's position from the 4 sec mark to the 7 sec. mark.

(b) When does the bat change direction?

at $x=5$, $x=7$, $x=10$



start 90ft Ents.

5sec 80ft →

← 10

(c) How far from the starting point is the bat after 6 seconds?

→ 80

10 ←

70ft from start.

(d) How far has the bat traveled in the 6 seconds?

→ 80

← 10

$$\begin{array}{r} 80 \\ +10 \\ \hline 90\text{ ft.} \end{array}$$

St 90 end.

5sec 50 →

← 20

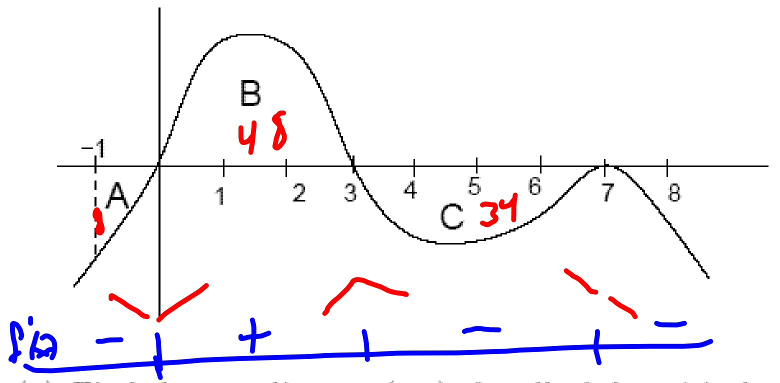
→ 50

(e) Does the bat make it out of the tunnel in 10 seconds?

$80 - 20 + 50 = 110$ ft

So yes.

2. The following is the graph of $f'(x)$. Use the fact that $f(0) = 40$ along with the given areas for the regions to answer the following. Region A = 8, Region B = 48, and Region C = 34.



(a) Find the coordinates, (x,y), for all of the critical values. Also classify the critical values.

Coord.

C.V. $x = 0, 3, 7$

$x = 0$ is a Local min $(0, 40)$

$x = 3$ is a. Local max $(3, 88)$

$x = 7$ is a neither. $(7, 54)$

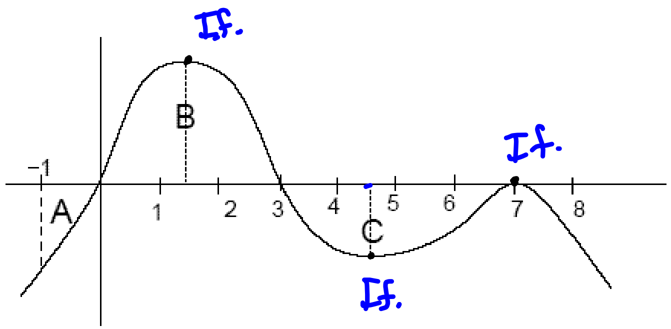
$$\int_0^3 f'(x)\,dx = f(3) - f(0)$$
$$48 = f(3) - 40$$
$$88 = f(3)$$

$$\int_3^7 f'(x)\,dx = f(7) - f(3)$$
$$-34 = f(7) - 88$$
$$54 = f(7)$$

(b) Find the x-values for the inflection points.

(c) Sketch the graph of $f(x)$.

2. The following is the graph of $f'(x)$. Use the fact that $f(0) = 40$ along with the given areas for the regions to answer the following. Region A $= 8$, Region B $= 48$, and Region C $= 34$.

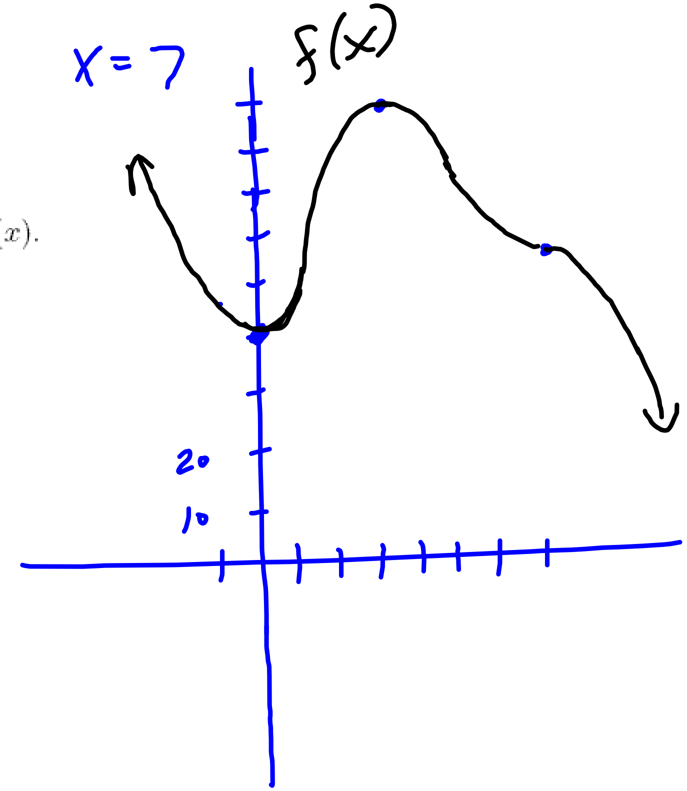


(b) Find the x-values for the inflection points.

$x \approx 1.5$

$x \approx 4.6$

$x = 7$



(c) Sketch the graph of $f(x)$.

$\int_{-1}^{0} f' = f(0) - f(-1)$

$-8 = f(0) - f(-1)$

$f(-1) = 40 + 8$

$f(-1) = 48$

3. If x is the number of years from 1990, then the population growth of a city, in millions per year, can be modeled by the formula $1.5e^{0.25x}$. $P'(x)$

(a) Write an expression that will give the growth of the city A years after 1990.

$$\int_0^A 1.5e^{.25x}\,dx = 1.5\left(\frac{1}{.25}\right)e^{.25x}\Big|_0^A$$

$$= 6e^{.25x}\Big|_0^A = 6e^{.25A} - 6e^0$$

$$= \boxed{6e^{.25A} - 6}$$

(b) If the city had a population of 5 million in 1990, find a formula that will give the poplation A years after 1990. $P(0)=5$

$$\int_0^A P'(x)\,dx = P(A) - P(0)$$

$$6e^{.25A} - 6 = P(A) - 5$$

$$\boxed{6e^{.25A} - 1 = P(A)}$$

4. If $f'(x) = 5\cos(3x^2)$ and $f(-1) = 2$ then find $f(1)$.

u-sub does not work on this function.

$$\int_{-1}^{1} 5\cos(3x^2)\,dx = f(1) - f(-1)$$

used fnInt

$$4.05955 = f(1) - 2$$

$$\boxed{6.05955 = f(1)}$$

5. Compute $\int_J^5 \frac{2}{x^2} + \cos(3x)dx$ $= \int_J^5 2x^{-2} + \cos(3x)\,dx$

$$= \frac{2x^{-1}}{-1} + \frac{1}{3}\sin(3x)\Big|_J^5$$

$$= -\frac{2}{x} + \frac{1}{3}\sin(3x)\Big|_J^5$$

$$= \left(-\frac{2}{5} + \frac{1}{3}\sin(15)\right) - \left(-\frac{2}{J} + \frac{1}{3}\sin(3J)\right)$$

6. (a) Compute $\int_0^A \frac{x}{1+x^2}\,dx = \int_{x=0}^{x=A} \frac{1}{2}\,\frac{1}{u}\,du$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\ln|u|\Big|_{x=0}^{x=A}$

$= \frac{1}{2}\ln(1+x^2)\Big|_0^A$

$= \frac{1}{2}\ln(1+A^2) - \frac{1}{2}\ln(1)$

0

$= \frac{1}{2}\ln(1+A^2)$

(b) Does $\int_0^\infty \frac{x}{1+x^2}\,dx$ converge or diverge?

since $\frac{1}{2}\ln(1+A^2) \to \infty$ as

$A \to \infty$

$\int_0^\infty \frac{x}{1+x^2}\,dx$ diverges.

$$\int_1^\infty \frac{1}{x^2}\,dx$$

$$\int_1^A x^{-2}\,dx = \left.\frac{x^{-1}}{-1}\right|_1^A = \left.-\frac{1}{x}\right|_1^A$$

$$= -\frac{1}{A} - \left(\frac{-1}{1}\right) = \frac{-1}{A} + 1$$

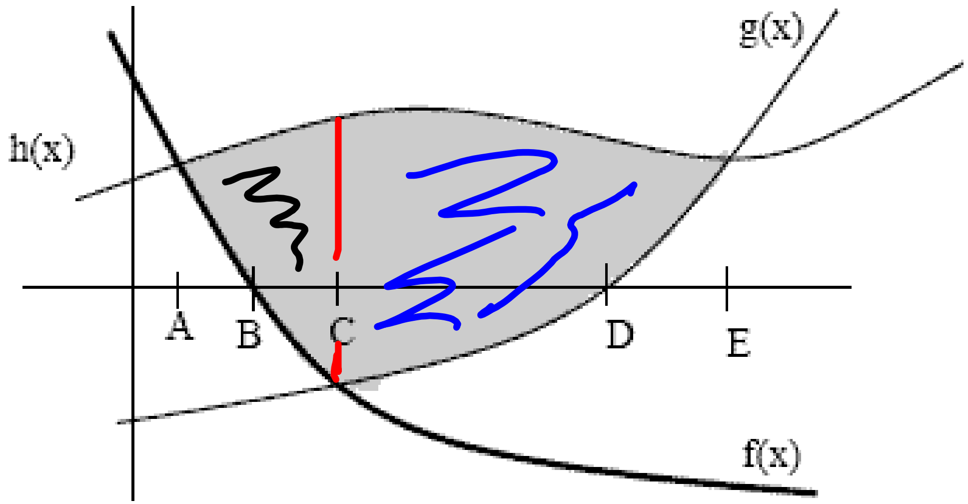
as $A \to \infty$ (gets very large)

$$-\frac{1}{A} \to 0$$

Means. $\int_1^\infty \frac{1}{x^2}\,dx = 1$

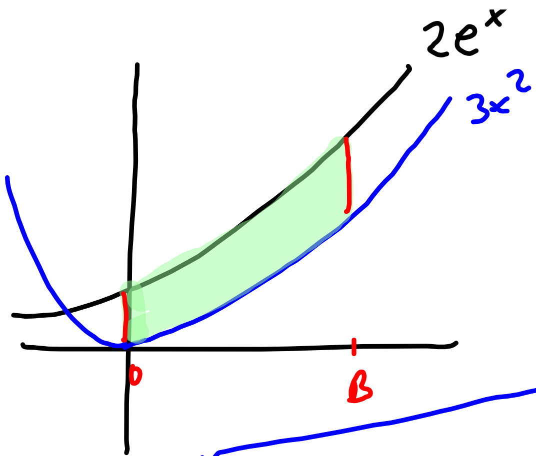
Say integral converges.

7. Set up the integral(s) that represent this shaded area.



$$\int_A^C h(x) - f(x)\,dx + \int_C^E h(x) - g(x)\,dx$$

8. Find the value of B so that the area between the curves $f(x) = 2e^x$ and $g(x) = 3x^2$ from $x = 0$ to $x = B$ will be 110.



$$\int_0^B 2e^x - 3x^2 \, dx = 110$$

$$\int_0^B 2e^x - 3x^2 dx = 2e^x - x^3 \Big|_0^B = \left(2e^B - B^3\right) - \left(2e^0 - 0\right)$$

$$= 2e^B - B^3 - 2$$

$$2e^B - B^3 - 2 = 110$$

$$\underbrace{e^B - B^3}_{y_1} = \underbrace{112}_{y_2}$$

only way to solve is by graphing on the calc.

$$\boxed{B = 4.6731}$$

9. Compute these integrals

(a) $\int \frac{1}{2x^3} - \sin(6x) + \sqrt[3]{x}\, dx = \int \frac{1}{2} x^{-3} - \sin(6x) + x^{1/3}\, dx$

$$= \frac{1}{2} \frac{x^{-2}}{-2} + \frac{1}{6}\cos(6x) + \frac{x^{4/3}}{\frac{4}{3}} + C$$

$$= -\frac{1}{4x^2} + \frac{1}{6}\cos(6x) + \frac{3}{4} x^{4/3} + C$$

(b) $\int \frac{e^{x^{-1}}}{x^2} dx$ $= \int -e^u \, du = -e^u + C$

$= \boxed{-e^{x^{-1}} + C}$

$u = x^{-1}$

$\frac{du}{dx} = -1x^{-2}$

$du = \frac{-1}{x^2} dx$

$-du = \frac{1}{x^2} dx$

(c) $\int \frac{5e^{5x} - 8e^{-8x}}{(e^{5x} + e^{-8x})^3} dx$ $= \int \frac{1}{u^3} du = \int u^{-3} du$

$$= \frac{u^{-2}}{-2} + C$$

$$= \boxed{-\frac{1}{2}\left(e^{5x} + e^{-8x}\right)^{-2} + C}$$

$u = e^{5x} + e^{-8x}$

$\frac{du}{dx} = 5e^{5x} - 8e^{-8x}$

$du = \left(5e^{5x} - 8e^{-8x}\right) dx$

OR

$$\boxed{-\frac{1}{2\left(e^{5x} + e^{8x}\right)^2} + C}$$

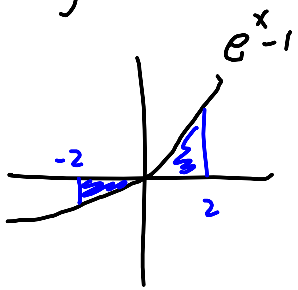
10. Compute the area between the x-axis and $y = e^x - 1$ from $x = -2$ to $x = 2$.

$$\int_{-2}^{2} e^x - 1 \, dx = \text{fnInt}(e^x - 1, x, -2, 2)$$
$$= 3.25372$$

This is <u>not</u> the answer.

$\int_{-2}^{2} e^x - 1 \, dx$ is the TOTAL Change in the Area.

you need to check the graph first.



$$\left| \int_{-2}^{0} e^x - 1 \, dx \right| + \int_{0}^{2} e^x - 1 \, dx$$

$$= \left| -1.135335 \right| + 4.389056$$

$$= 5.524391$$

$$\text{base} = \frac{9-1}{4} = \frac{8}{4} = 2$$

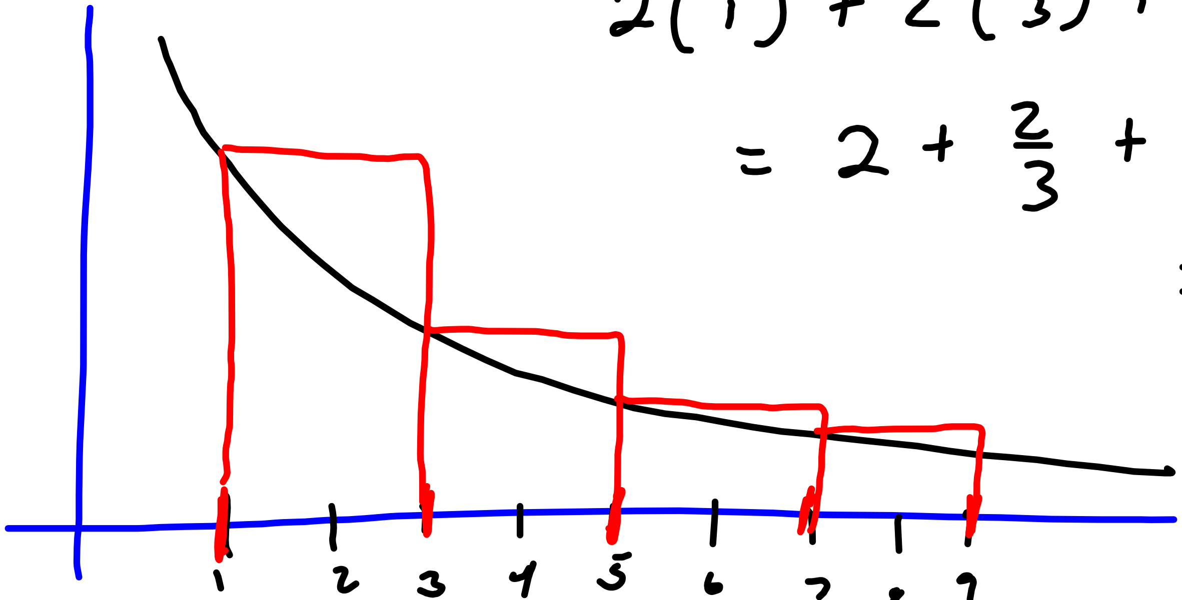
11. (a) Estimate $\displaystyle\int_1^9 \frac{1}{x}dx$ using a left-hand sum with 4 rectangles.

$f(x) = \frac{1}{x}$

$$2\left(\frac{1}{1}\right) + 2\left(\frac{1}{3}\right) + 2\left(\frac{1}{5}\right) + 2\left(\frac{1}{7}\right)$$

$$= 2 + \frac{2}{3} + \frac{2}{5} + \frac{2}{7}$$

$$= 3.35238$$



(b) Is the estimate an over estiamte or an under estimate? Justify your answer with a graph.

over since Rect. go above graph of $\frac{1}{x}$

12. Estimate $\int_1^{10} f(x)dx$ using a right sum and the information in the table.

x	1	4	8	10	11
f(x)	5	20	16	8	3

$$3(20) + 4(16) + 2(8)$$

if you need graph the points