

Extra determinant problems

Problem 1. Let A be the 3×3 matrix:

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

and consider the matrix

$$A - xI \equiv \begin{pmatrix} 2-x & -1 & 0 \\ -1 & 2-x & -1 \\ 0 & -1 & 2-x \end{pmatrix}.$$

Compute the determinant of $A - xI$ (it is a cubic polynomial in x). Compute its roots.

Problem 2. Let A be the 10×10 matrix defined by

$$A_{ij} = \begin{cases} 2: & \text{if } i = j, \\ -1: & \text{if } j = i + 1, \\ 0: & \text{otherwise.} \end{cases}$$

Note that A is an upper triangular matrix.

- Compute the determinant of A .
- Compute the determinant of $A - xI$ (a polynomial in x of order 10).