Problem 1 *Find* $\gcd(1234, 321)$.

Problem 2 *If* $ab \equiv cd \mod n, b \equiv d \mod n$ *and* $\gcd(b, n) = 1$, *then* $a \equiv c \mod n$.

Problem 3 *Show that* 5 *divides* $98^{150} + 3(103)^{151}$.

Problem 4 *If* p *and* q *are distinct primes and* $a, b \in \mathbb{N}$ *are given with the property that*

$$a^p \equiv b \text{ (mod } q\text{) and } a^q \equiv b \text{ (mod } p\text{)} ,$$

then show that

$$a^{pq} \equiv b \text{ (mod } pq\text{)} .$$

Problem 5 *What is the last digit of* 17^{3241}*?*

Problem 6 *Prove that: If* a *and* b *are integers are* p *is a prime, then* $(a + b)^p \equiv a^p + b^p \mod p$.

Problem 7 *Prove:* $5\big|(58^{13} + 3(13)^{58})$.

Problem 8 *Assume* $a, b, c, d, e \in \mathbb{N}$. *If a natural number,* x, *is written as* $x = 10^8 * a + 10^6 * b + 10^4 * c + 10^2 * d + e$, *show that* 99 *divides* x *if and only if* 99 *divides* $a + b + c + d + e$.

Problem 9 *If* p *and* q *are distinct primes and* a, b *and* $c \in \mathbb{N}$ *are given with the property that*

$$a^p \equiv b \text{ (mod } q\text{)} , c^q \equiv b \text{ (mod } p\text{) and}$$
$$a \equiv c \mod pq,$$

then show that

$$a^{pq} \equiv b \mod pq$$

Problem 10 *What is the last digit of* 13^{4231}.

Problem 11 *Assume* $a, b, c, d, e \in \mathbb{N}$. *If a natural number,* x, *is written as* $x = 10^{12} * a + 10^8 * b + 10^4 * c + d$, *show that* 101 *divides* x *if and only if* 101 *divides* $a + b + c + d$.