Problem 1 *Prove that the remainder when dividing* 37^{217} *by* 11 *is* 5.

Problem 2

(a) Prove that 743 *and* 249 *are relatively prime.*

(b) Find $x, y \in \mathbb{Z}$ *such that* $743x + 249y = 1$

Problem 3

(a) Find the last ("the ones") digit of 543^{345}. **Note:** 10 **is not a prime.**

(b) If a number n *is written as* $n = a_1 7^k + a_2 7^{k-1} + a_3 7^{k-2} + \cdots + a_k 7 + a_{k+1}$, *we say that it is written in base* 7. *The last digit in base* 7 *would be* a_{k+1}. *Show that the last digit in base* 7 *of* 851^{177} *is* 1.

Problem 4 *Prove that for any prime* p *and any* $a \in \mathbb{N}$, *which is relatively prime to* p, *the two sets*

$$\{r_1, r_2, \ldots, r_{p-1}\} \text{ and } \{1, 2, 3, \cdots, (p-1)\}$$

are the same, where r_j *is the remainder (a number from* 1 *to* $p-1$*) when* ja *is divided by* p.