Rules of the Game.

(a) Give complete explanations for your answers.

(b) Ask if you have a question.

(c) Never give an answer which trivializes the problem.

The Problems.

Problem 1. For the set $A \subseteq U$, if $\mathcal{P}(A) = \mathcal{P}(A^c)$, then $A = \emptyset$.
Problem 2. For $A \subseteq U$, if $\mathcal{P}(A) \bigcup \mathcal{P}(A^c) = \mathcal{P}(U)$, then either $A = U$ or $A = \emptyset$.
Problem 3. Prove the following. $A \cup B = A \cup (B \cap A^c)$, by checking that each side is contained in the other.
Problem 4. Prove using the rules of inference that
(a) $((a \to b) \wedge (a \to c)) \Longleftrightarrow (a \to b \wedge c)$.
(b) Use part (a) to show $((p \to q) \wedge (p \to \neg q)) \Longleftrightarrow \neg p$.
Problem 5. Prove that $(2n)! \leq 2^{2n}(n!)^2$.
Problem 6. Let $a_0 = 1,\ a_1 = 2,\ $ and $a_2 = 3$. For $n \geq 1$, define

$$a_{n+3} = a_n + a_{n+1} + a_{n+2}.$$

Show that $a_n \leq 2^n, \forall n \geq 1$.
Problem 7. Prove by induction that $\sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j = \left(\sum_{k=1}^{n} a_k\right)^2$.
Problem 8. Assume that $f : Y \longrightarrow X$. Define $\rho : X \longrightarrow \mathcal{P}(Y)$ by $\rho(x) = f^{-1}(\{x\})$. Show that if f is onto, then ρ is 1-1.