Problem 1 Find $\gcd(1234, 321)$.

Problem 2 If $ab \equiv cd \mod n, b \equiv d \mod n$ and $\gcd(b, n) = 1$, then $a \equiv c \mod n$.

Problem 3 Show that 5 divides $98^{150} + 3(103)^{151}$.

Problem 4 If $p$ and $q$ are distinct primes and $a, b \in \mathbb{N}$ are given with the property that

$$a^p \equiv b \pmod{q} \text{ and } a^q \equiv b \pmod{p},$$

then show that

$$a^{pq} \equiv b \pmod{pq}.$$

Problem 5 What is the last digit of $17^{3241}$?

Problem 6 Prove that: If $a$ and $b$ are integers are $p$ is a prime, then

$$(a + b)^p \equiv a^p + b^p \mod p.$$

Problem 7 Prove: $5|(58^{13} + 3(13)^{58})$.

Problem 8 Assume $a, b, c, d, e \in \mathbb{N}$. If a natural number, $x$, is written as

$$x = 10^8 \ast a + 10^6 \ast b + 10^4 \ast c + 10^2 \ast d + e,$$

show that 99 divides $x$ if and only if 99 divides $a + b + c + d + e$.

Problem 9 What is the last digit of $13^{4231}$?

Problem 10 Assume $a, b, c, d, e \in \mathbb{N}$. If a natural number, $x$, is written as

$$x = 10^{12} \ast a + 10^8 \ast b + 10^4 \ast c + d,$$

show that 101 divides $x$ if and only if 101 divides $a + b + c + d$. 
