Math 220, Zinn Practice
Name.

Problem 1 *Prove that for any proposition, p ,*

$$(p \Rightarrow \neg p) \Rightarrow ([p \wedge (\neg p)] \vee (\neg p))$$

Problem 2 *Let U denote the universe and A and B two subsets. Prove that if $A \cup B = A^c \cup B$, then $B = U$.*

Problem 3 *On a certain island there are three types of people: knights, who always tell the truth; knaves, who always lie; and normal people, who sometimes lie and sometimes tell the truth. We are given three people on this island, A, B and C, one of whom is a knight, one is a knave, and one normal (but not necessarily in that order). They make the following statements:*

A: I am not normal.
B: That is true.
C: I am normal.

What are A, B and C?

Problem 4 *Show that $2^{1/3}$ is irrational. You may need to prove a lemma first.*

Problem 5 *Prove that if A, B are sets in the universe, U, then*

$$(A \backslash B) \cup (B \backslash A) = (A \cup B) \backslash (A \cap B).$$

Problem 6 *Let $\mathcal{P}(A)$ be a symbol for the collection of all subsets of A. For example, if $A = \{1, 2, 3\}$, then $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.*

Show that

(a) $A \subseteq B$ implies $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

(b) Prove or give a counterexample to: if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$.

Problem 7 *Let $n \in \mathbb{N}$ have the property that: if $a, b \in \mathbb{N}$ and n divides ab, then either n divides a or n divides b. Show that n is a prime.*