More Practice for Exam 1

Problem 1. Using the Archimedean property show that for every positive number ϵ there is a natural number N such that for every natural number $n \geq N, \frac{1}{n} < \epsilon$.

Problem 2. Using the Archimedean property show that every positive real number has a positive square root.

Problem 3. (a) If $A = [a, b]$ is a closed interval in $\mathbb{R}$, then $\sup(A) = b$. (b) If A, B are closed intervals in $\mathbb{R}, \sup(A) = \sup(B)$ and $\inf(A) = \inf(B)$, then $A = B$.

Problem 4. If for each $j \geq 1$, the set A_j is countably infinite and these sets are disjoint, prove that the union is countably infinite.

Problem 5. Suppose that U is a set and d is a metric on U. Further assume that $f : [0, \infty) \longrightarrow [0, \infty)$ is increasing, $\frac{f(t)}{t}$ is **non-**increasing and $f(0) = 0$. Define $\rho(x, y) = f(d(x, y))$. Show that ρ is a metric.

Problem 6. Let $A_j, j = 1, \ldots$ be a sequence of subsets of $\mathbb{R}_+$ with $\sup(A_{j+1}) < \inf(A_j)$, then $\inf(\bigcup_{j=1}^{\infty} A_j)$ is a limit point of $\bigcup_{j=1}^{\infty} A_j$, which is not in $\bigcup_{j=1}^{\infty} A_j$.

Problem 7. Let (U, d) be a metric space. Define a set to be **clopen** if it is both closed and open. If each of $A_j, j = 1, \ldots, n$ are clopen, show that $\bigcup_{j=1}^{n} A_j$ is clopen.