Math 409H, J. Zinn **Practice for Exam 1**

Some of these problems you will already have seen, but they are examples of test questions.

1. **If A and B are non-empty subsets of $\mathbb{R}$ that are bounded above, and write**

$$A + B = \{a + b : a \in A, b \in B\}.$$

   Prove that $A + B$ is bounded above and $\sup(A + B) = \sup(A) + \sup(B)$.

2. **Find the infimum and supremum of**
   (a) $E = \{x \in \mathbb{R} : x^2 - 3x - 5 = 0\}$.
   (b) $E = \{x \in \mathbb{R} : x^2 - 3x - 5 > 0\}$.

3. **Given two subsets A and B of $\mathbb{R}$. Assume that $\sup(A) < \inf(B)$. Show that for each $a \in A$ and $b \in B$ we have $a < b$.**

4. (a) **Prove that if E is uncountable and F is countable, then $E \setminus F := E \cap F^c$ is uncountable.**
   (b) **Show that the set $S = \{m + n : m, n \in \mathbb{N}\}$ is countable.**

5. **In a metric space a set is closed if and only if its complement is open.**

6. **In a metric space every neighborhood is an open set.**

7. **Show that the set of all limit points of a set E in a metric space (X, d) is a closed set. (Of course, this is part of 43/6.)**

8. **44/10.**

9. **45/22**