Math 142 Week In Review
Problem Set #10
Review for Exam 3 (5.4 – 7.2)
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Section 7.1

1. Find the area of the region bounded by $f(x) = 4 - x^2$ and $g(x) = -5$.

2. Find the area of the region bounded $f(x) = x^2 - 2$ and $y = 2$ on [-3, 2].

3. As part of the study of the effects of World War II on the economy of the United States, and economist used data from the U.S. Census Bureau to produce the following Lorenz curves for the distribution of income in the United States in 1935 and in 1947.

$$f(x) = x^{2.4} \quad \text{(Lorenz curve for 1935)}$$

$$f(x) = x^{1.6} \quad \text{(Lorenz curve for 1947)}$$

Find the Gini index of income concentration for each Lorenz curve and interpret the results.

Section 7.2

4. Just Smell It, a scented candle shop, has determined that the demand function for a large vanilla-scented candle is given by $d(x) = 20 - \frac{1}{2}x$ while the related producer supply function is given by $s(x) = \frac{1}{2}\sqrt{x}$, where x is the monthly quantity and $d(x)$ and $s(x)$ are in dollars per candle.

   a) Determine the equilibrium point. Round the equilibrium demand to the nearest whole number and round the equilibrium price to the nearest dollar.
   (b) Determine the consumers' surplus at equilibrium.
   (c) Determine the producers' surplus at equilibrium.
   (d) Construct a graph of the supply and demand curves, then shade the region of the graph that represents the company's revenue.

5. Find the total income produced by a continuous income stream in the first 3 years if the rate of flow is $f(t) = 400e^{0.05t}$.

Sections 5.4-6.5

6. If $f(x)$ is a continuous function with domain $(-\infty, \infty)$, $f`(x) = \frac{2 + 0.225x^{7/3}}{3x^{1/3}}$,

and $f''(x) = \frac{-2 + 1.35x^{7/3}}{9x^{4/3}}$ use derivative techniques to find:

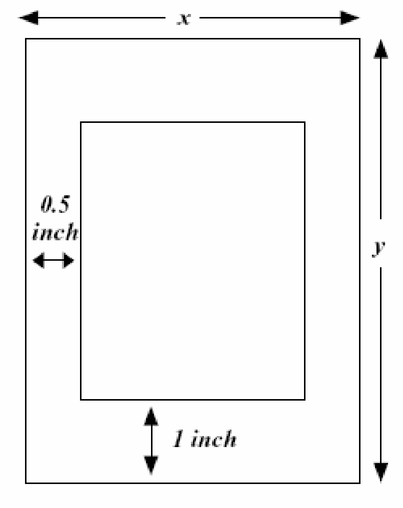
   (a) the intervals where f(x) is increasing and decreasing.
   (b) relative extrema of f(x).
   (c) the intervals where f(x) is concave up and concave down.
   (d) all points of inflection.
   (e) a possible graph of f(x).

7. Find the absolute extrema of $f(x) = x^3 + 6x^2 + 9x$ on

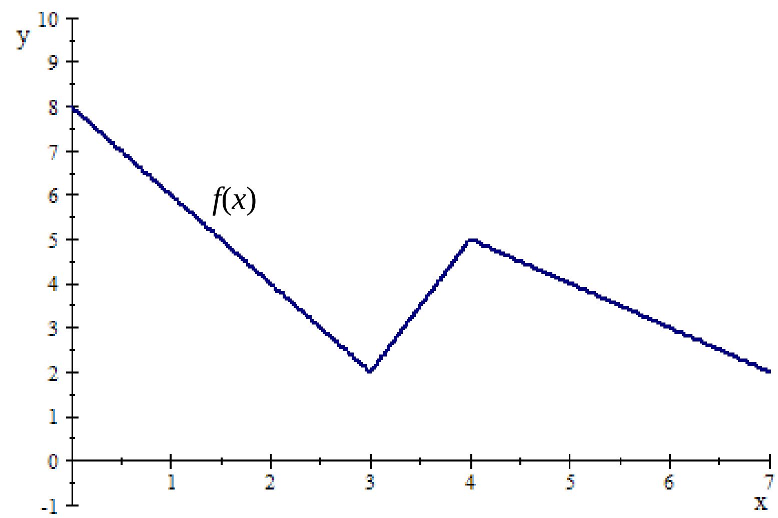
   (a) [−5, −1] (b) (−2, 2)

8. A fence is to be built to enclose a rectangular area adjacent to a building. The building, 60 feet long, will be used as part of the fencing on one side of the area. Find the dimensions that will enclose the largest area if 360 feet of fencing material is used.

9. A book designer has decided that the pages of a book should have 1-in margins at the top and bottom and 0.5 in margins on the sides. She further stipulated that each page should have an area of 50 square inches. Determine the page dimensions that will result in the maximum printed area on the page.



10. Use the graph of $f(x)$ below to answer questions a and b.



a) Approximate $\int_1^4 f(x)\,dx$ using 3 right hand rectangles and 3 left hand rectangles.

b) Compute the $\int_1^4 f(x)\,dx$ using geometric formulas.

11. Evaluate the following integrals.

a) $\int_1^2 \frac{2x-7}{x^3}\,dx$ b) $\int_a^b 4x-2\,dx$ c) $\int 12t^2(t^3+5)^4\,dt$ d) $\int \frac{4x+10}{x^2+5x-3}\,dx$

e) $\int x\sqrt[3]{x-2}\,dx$ f) $\int 3xe^{x^2+1}\,dx$ g) $\int \frac{e^{2x}}{e^{2x}+1}\,dx$ h) $\int \frac{5}{\sqrt{x}\left(\sqrt{x}+4\right)}\,dx$

12. Set up the following integral using u-substitution.

a) $\int_0^1 \frac{x}{(2x^2+1)^3}\,dx$ b) $\int_1^{e^2} \frac{(\ln(x))^2}{x}\,dx$

13. A company determines that the marginal profit function for selling a new kind of football is $P`(x) = 0.0002x^2 - 0.02x + 2.85,\ 0 \le x \le 100$ where $x$ is the number of footballs made and sold and $P`(x)$ is the marginal profit function in dollars per football.

a) Knowing that the company breaks even if 20 footballs are sold, recover the profit function $P(x)$.

b) Evaluate $\int_{50}^{100} P`(x)\, dx$ and interpret.

14. Find the average value of $f(x) = 6x^2 + 2x$ over the interval [–1, 2].