Math 142 Week In Review
Problem Set #8
(5.4, 5.5, 5.6, 6.1)
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Section 5-4

1. Sketch a graph of a function that satisfies the following conditions:
   - Domain: $(-\infty,\infty)$
   - Range: $[2,\infty)$
   - Continuous on $(-\infty,\infty)$
   - $f'(x) > 0$ on $(-3,0) \cup (3,\infty)$
   - $f'(x) < 0$ on $(-\infty,-3) \cup (0,3)$
   - $f`(0)$ is undefined
   - $f''(x) > 0$ on $(-\infty,0) \cup (0,\infty)$
   - $\lim_{x\to\infty} f(x) = \infty$
   - $\lim_{x\to-\infty} f(x) = \infty$
   - $f(-3) = 2,\ f(0) = 5$

2. Given $f(x)=\dfrac{x^3-5x^2+6x}{x^2-x-2}$, summarize all pertinent information obtained by applying the graphing strategy, and sketch the graph of $y = f(x)$. (Hint: $f\,'(x)=\dfrac{x^2+2x-3}{(x+1)^2}$ and $f\,''(x)=\dfrac{8}{(x+1)^3}$.)

Section 5-5

3. Find the absolute maximum and minimum, if either exists, for $f(x) = x^3 - 12x^2 + 36x - 15$ on the interval

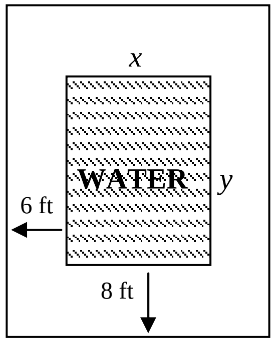
   a) [1, 9]
   b) [1, 4]
   c) (3, 7)

4. Find the absolute extrema for $f(x)=\frac{e^x}{x^2}$ on the interval $(0,\infty)$.

5. Find the absolute extrema for $f(x) = x^3(\ln(x) - 2)$ on the interval $(0, \infty)$.

Section 5-6

6. Your friend's family is planning to build a rectangular swimming pool. The water's surface is to have an area of 1200 square feet. The pool will also have a 6 foot wide walkway on the east and west sides and an 8 foot walkway on the north and south sides. Let x be the length of the pool (east to west) and let y be the width of the pool (north to south). Given that $10 \le x \le 100$, find the dimensions that will minimize the total area of the swimming pool and walkways.



7. A closed box is made with a square base and must have a volume of 343 cubic inches. The material for the sides and the top costs \$0.02 per square inch, and the material for the base costs \$0.04 per square inch. Determine the dimensions of the box that minimizes the cost of the materials.

8. The owner of a luxury motor yacht that sails among the 4000 Greek islands charges $600 per person per day if exactly 20 people sign up for the cruise. However, if more than 20 people sign up for the cruise, the each fare is reduced by $4 for each additional passenger. Assuming at least 20 people sign up for the cruise, determine how many passengers will result in the maximum revenue for the owner of the yacht. What is the maximum revenue? What would be the fare per passenger in this case?

Section 6.1

9. Determine if $F(t) = 7t + et + C$ is the antiderivative of $f(t) = 7 + e$.

10. Find the following.

a) $\int x^4 \, dx$

b) $\int \left(4t^3 + 5t - 6\right) dt$

c) $\int \left(\sqrt[3]{x^2} - 3x^{1/4}\right) dx$

d) $\int \left(\frac{4p^8 - 5p^3}{p^6}\right) dp$

11. Find $F(t)$ such that $F'(t) = \frac{1 - t^4}{t^3}$ and $F(1) = 4$.

12. Find the antiderivative of $\frac{dy}{dx}\frac{5x+2}{\sqrt[3]{x}}$ given $y(1) = 0$

13. Find the antiderivative of $\frac{dx}{dt}4e^{t}-2$ given $x(0) = 1$.

14. The daily marginal revenue function for BlackDay Sunglass Company is given by $R'(x)=30-0.0003x^{2},\ 0\le x\le 540$, where x represents the number of sunglasses produced and sold. Knowing that $R(x) =$ 1487.5 when $x = 50$, recover the revenue function R.