

**MATH 142 Week in Review**  
**Problem Set #12**  
**Review for Final Exam**

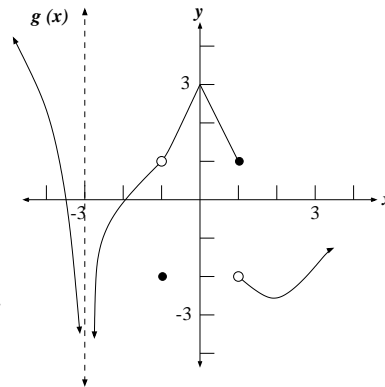
- Suppose that 10,000 units of a certain item are sold per day by the entire industry at a price of \$150 per unit and that 4000 units can be sold per day by the same industry at a price of \$300 per unit. Find:
  - the demand equation for  $p$ , assuming the demand curve to be linear.
  - the revenue equation
  - the number of items that need to be sold per day to maximize revenue.
  - the price of the item that will give maximum revenue.
  - the producers' surplus given the supply equation to be  $p = 0.05x + 100$
- The table below shows the relationship between the temperature and the number of eggs laid by the female citrus rust mite. Find the best mathematical model to predict the number of eqqs a female citrus rust mite would lay if the temperature is 37 degrees Celsius.

$x$ temperature in ° C	14	17	19	21	23	25	27	29	31
$y$ # eggs per female	2.22	6.10	7.93	14.69	11.52	10.55	15.44	11.58	11.01

- Find the domain of the following functions:
  - $f(x) = \frac{x+3}{x^2+3x}$
  - $f(x) = \frac{\sqrt{x+5}}{x+1}$
  - $f(x) = \ln(2x+1)$
- Use properties of logarithms to write  $\log_3 \frac{4\sqrt{3}}{9}$  as a sum/difference of logarithms.
- Solve  $4.9 = 2.05 + 1.3 \ln x$  for  $x$ .
- Determine how much money is in an account after 8 years if \$1000 is initially deposited at 6% interest, compounded quarterly.
- Determine the average value of an account over 40 years if \$500 is initially deposited at 5% interest, compounded continuously.
- At the beginning of each month, Susan deposits \$250 into her 403-b plan, which earns 6.5% annual interest compounded continuously. Determine how much Susan will have in her account after 30 years.
- Find the limits if they exist
  - $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3}$
  - $\lim_{x \rightarrow 2} \frac{x^2 + 1}{x - 2}$
  - $\lim_{x \rightarrow \infty} (1 + 2e^{-x})$
- Find all asymptotes of the following functions.
  - $f(x) = \frac{2x^3 + x - 3}{x^3 + 3}$
  - $f(x) = \frac{x^2 + 1}{3x(x^2 - 1)}$
- Use the limit definition of derivative to find the derivative of  $f(x) = \frac{1}{x}$

12. Use the graph at right to find

- (a)  $\lim_{x \rightarrow -1} g(x)$   
 (b)  $\lim_{x \rightarrow 0} g(x)$   
 (c)  $\lim_{x \rightarrow -3} g(x)$   
 (d) Find all points where  $g(x)$  is discontinuous.  
 (e) Find all points where  $g(x)$  does not have a derivative.



13. Find the equation the line tangent to  $f(x) = \frac{1}{x^3} + 1$  at  $x = 2$ .
14. Find the derivatives of the following functions  
 (a)  $f(x) = 4^{2x^3+4x-1}(x^2 + x + 2)^5$       (b)  $f(x) = \frac{(x^2 + 3)^4}{e^{3x^2}}$       (c)  $f(x) = \ln\left(\frac{2x^2 + 1}{x - 4}\right)$
15. Given the cost equation  $C(x) = \sqrt{x}(x + 12) + 50$  where  $x$  is the number of items produced and  $C(x)$  is the cost, in dollars, of producing  $x$  items. Find the approximate cost of producing the 36th item.
16. Find the critical points for  $f(x) = x + \ln x$
17. Given  $f'(x) = (x - 4)^2(x - 1)(x + 3)$  find the intervals where  $f(x)$  is increasing and decreasing.
18. Given  $g(x) = 2x + \frac{18}{x}$ , find all relative extrema and points of inflection.
19. Find the absolute extrema of  $2x^3 - 3x^2 + 1$  on  $[-1, 2]$ .
20. A closed box with a square base is to have a volume of  $16000 \text{ cm}^3$ . The material for the top and bottom of the box costs \$3 per square cm, while the material for the sides costs \$1.50 per square centimeter. Find the dimensions of the box that will lead to minimum total cost.
21. Find the following.  
 (a)  $\int_1^3 \frac{\sqrt{\ln x}}{x} dx$       (b)  $\int_{-2}^3 (-x^2 - 3x + 5) dx$       (c)  $\int x e^{x^2+5} dx$
22. De Win Enterprises has found that its expenditure rate per day (in hundreds of dollars) on a certain type of job is given by  $E'(x) = 4x + 2$ , where  $x$  is the number of days since the start of the job.
- (a) Find the total expenditure if the job takes 10 days.  
 (b) How much will be spent on the job from the tenth to the twentieth day?  
 (c) If the company wants to spend no more than \$5000 on the job, in how many days must they complete it?
23. Find the area enclosed by  $y = |x|$  and  $y = \sqrt[3]{x^2}$  on  $[-8, 10]$ .
24. The Kelomata Company determines that the daily cost of producing patio swings can be modeled by  $C(x) = 15,000 + 100x - 0.001x^2$  where  $x$  represents the number of patio swings produced each day and  $C(x)$  represents the daily production cost in dollars. Find the marginal average cost function.

25. The Digital Calculator Company manufactures scientific calculators and graphing calculators. Let  $x$  represent the weekly demand for scientific calculators, and let  $y$  represent the demand for graphing calculators. The weekly price-demand equations are given by

$$p = 60 - 0.3x + 0.1y, \quad \begin{array}{l} \text{the price in dol-} \\ \text{lars for a scien-} \\ \text{tific calculator} \end{array} \quad q = 100 + 0.2x - 0.5y, \quad \begin{array}{l} \text{the price in dol-} \\ \text{lars for a graph-} \\ \text{ing calculator} \end{array}$$

The cost function is given by  $C(x, y) = 3000 + 25x + 35y$ .

- (a) Find the weekly revenue function  $R(x, y)$ .  
 (b) Find the weekly profit function  $P(x, y)$ .  
 (c) Evaluate and interpret  $P(40, 60)$ .  
 (d) Evaluate and interpret  $P_x(40, 60)$ .
26. Find the indicated first order partial derivatives.  
 (a)  $f_x(x, y) = \sqrt{1 + x^4y^3}$       (b)  $f_y(x, y) = xye^{x-y^2}$       (c)  $f_x(x, y) = e^{y^2} \ln(yx^2 + xy^2)$
27. Find all relative extrema and saddle points for  $f(x, y) = x^2 - 2xy + \frac{1}{3}y^3 - 3y$ .