Math 172H **PRACTICE FOR EXAM 1** **Name:**__________________________

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1. Compute $\int x^2 \ln^2 x \, dx + \frac{2}{3} \int x^2 \ln x \, dx$.
   (a) $\frac{1}{3}x^3 \ln^2 x$ (b) $\frac{2}{3}x^3 \ln^2 x$ (c) $\frac{1}{3}x^3 \ln x$ (d) $\frac{2}{3}x^3 \ln x$ (e) $x^3 \ln^2 x$

2. Compute $\int \tan^2 x \sec^4 x \, dx$.
   (a) $\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x$ (b) $\frac{2}{5}\tan^5 x + \frac{1}{3}\tan^3 x$ (c) $\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^2 x$ (d) $\frac{2}{5}\tan^5 x + \frac{1}{3}\tan^2 x$
   (e) $\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^4 x$

3. The base of the solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are equilateral triangles. Find the volume.
   (a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $3\sqrt{3}$ (d) $4\sqrt{3}$ (e) $5\sqrt{3}$

4. The region bounded by the curve $y = \sqrt{\cos x}$, $0 \leq x \leq \dfrac{\pi}{2}$, $y = 0$ and $x = 0$ is rotated about the x-axis. Find the volume
(a) 1 (b) π (c) 2 (d) 2π (e) 3

5. If one writes $\dfrac{x^2+x+2}{(x^2+1)(x-1)}$ in partial fractions as $\dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1}$, then (A, B, C) are
(a) $(1, 1, 0)$ (b) $(2, 1, 0)$ (c) $(2, -1, 1)$ (d) $(2, -1, 0)$ (e) $(1.0, 1)$

6. Let S be the solid whose base is the region enclosed by the curve $y = \sqrt{x}$, the straight line $x + y = 2$, and the x-axis. The cross sections of S perpendicular to the x-axis are semicircles. Compute the volume of S.

   (a) (i) Obtain the equation of the tangent line to the curve $y = x^3$ at the point $(1, 1)$. Determine the co-ordinates of the point where this tangent line meets the y-axis.
   (b) (ii) Let $\mathcal{R}$ denote the region enclosed by the curve $y = x^3$, the tangent line to this curve at the point $(1, 1)$, and the y-axis. Compute the area of $\mathcal{R}$.
   (c) (iii) Use the method of shells to compute the volume of the solid obtained by rotating the region $\mathcal{R}$ (given above) about the line $x = -1$.

7. Let $\mathcal{R}$ be the region enclosed by $y = \ln(x)$, the x-axis, $x = 1$, and $x = e$. Employ the method of discs to calculate the volume of the solid obtained by rotating $\mathcal{R}$ about the x-axis.

8. Evaluate each of the following integrals:

   (a) (i) $\displaystyle\int t(t-1)^{99}(t+1)^{99}\, dt$

   (b) (ii)) $\displaystyle\int_0^{\pi/2} \cos^8(\theta)\, d\theta$

   (a) (iii) $\displaystyle\int \tan^7(x)\sec^5(x)\, dx$

   (b) (iv) $\displaystyle\int \sin^2(x)\sin(4x)\, dx$

9. (a) (i) State the theorem on Integration by Parts.

   (b) (ii) Suppose that f is twice differentiable in $[-1,1]$, and that f'' is continuous throughout that interval. Given that

$$f(-1) = f(1) = \int_{-1}^{1} f(x)\,dx = 1,$$

show that

$$\int_{-1}^{1} (1-x^2) f''(x)\,dx = 2.$$

10. Find the area between the curves $y = 2\sin x$ and $y = \sin 2x$ from $0 \le x \le \pi$.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

11. How large should we take n in order to guarantee that when using Simpson's Rule to approximate $\int_2^4 x \ln x\,dx$, the estimate is accurate to within $\dfrac{1}{45(2^{11})}$?

(a) 2 (b) 4 (c) 6 (d) 8 (e) 10

12. Find the area of the surface generated by revolving the curve $y = \sqrt{2x - x^2}$ from $x = 0$ to $x = 2$ about the x-axis.

(a) π (b) $\dfrac{3}{2}\pi$ (c) $\dfrac{5}{2}\pi$ (d) 3π (e) 4π

13. Solve the equation $x\frac{dy}{dx} - 2y = x^3 \sec x \tan x$, when $y(\pi) = 0$. Then find $y(\frac{\pi}{3})$.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{3}$ (e) $\frac{\pi^3}{4}$