

1. Show that $|\sin(nx)| \leq n|\sin(x)|$ for all x .
 2. Show that 36 divides $7^n - 6n - 1$ for all $n \in \mathbb{N}$.
 3. Show that $n! > n^2$ for all $n \geq 4$.
 4. Show that $\sqrt{17}$ is not rational.
 5. Show that $|\sum_{i=1}^n a_i| \leq \sum_{i=1}^n |a_i|$ for any real numbers $\{a_i\}_{i=1}^n$.
 6. Show that if a set E has two or more elements, then $\inf E < \sup E$.
 7. Using an ϵ argument, show that $\lim_{n \rightarrow \infty} \frac{3n+1}{7n-4} = \frac{3}{7}$.
 8. Consider $s_n = \sqrt{n^2+n} - n$.
 - (a) Guess the limit.
 - (b) Prove your answer is correct using an ϵ proof.
 9. Assume that the sequence $\{x_n\}$ converges to x and that none of the x_n or x are zero. Show that the sequence $\{\frac{1}{x_n}\}$ is bounded below.
- Some additional problems.**
10. Prove: $\{a_n\}$ bounded implies $\{a_n^2 + a_n + 2\}$ bounded.
 11. Show that $n^{1/\sqrt{n}}$ converges to 1.
 12. Let $a_1 = 2$ and for $n \geq 2$, $a_n = 1/2(a_{n-1} + \frac{2}{a_{n-1}})$. Show that (i) $a_n \geq \sqrt{2}$ (This is a bit tricky), (ii) a_n decreases and (iii) $a_n \rightarrow \sqrt{2}$.