

1. Show that $2^x = 3x$ has at least one real solution.
2. Prove that every cubic polynomial, $f(x) = x^3 + bx^2 + cx + d$, has at least one root.
3. Use the Theorems in the book to prove that $f(x) = x \sin^2 x + \frac{1}{x}$ is continuous at x in $[1, 2]$.
4. Use the limit theorems proved in the book to show that $f(x) = xe^x + x^2$ is continuous.
5. Let $\{b_n\}$ be a sequence of positive numbers converging to zero. Suppose that the terms of a sequence $\{a_n\}$ satisfy

$$|a_m - a_n| \leq b_n \text{ for all } m \geq n.$$

Prove that $\{a_n\}$ is a Cauchy sequence.

6. Let $\{a_n\}$ be an increasing sequence, which is also bounded. Define $\{b_n\}$ by

$$b_n = \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

Prove that $\{b_n\}$ is monotone and bounded and therefore has a limit.

7. Use the fact that $\max(a, b) = \frac{a + b + |a - b|}{2}$ to show that if both f and g are functions which are continuous at $x = a$, then the function h defined by

$$h(x) := \max(f(x), g(x))$$

is also continuous at $x = a$.

8. Let f and g be continuous functions on the finite interval $[a, b]$. Suppose that $f(x) < g(x)$ for all x in $[a, b]$. Prove that there is an $0 < \alpha < 1$ such that $f(x) \leq \alpha g(x)$ for every x in $[a, b]$.
9. There could also be definitions and proofs of Theorems in the book.