1. If X has density

$$f(x) = \begin{cases} cx^3 e^{-x}) & ,0 \le x < \infty \\ 0 & , \text{ otherwise .} \end{cases}$$

Find the value of c that makes f a probability density.

2. Let X have density

$$f(x) = \begin{cases} xe^{-x} & ,0 < x < \infty \\ 0 & , \text{ otherwise .} \end{cases}$$

(a) Find $P(X > 2)$.

(b) Find $F(t)$

(c) Find $E(X)$.

3. If the distribution function of X is given by $F(0^-) = 0, F(0) = 1/4$, F linear between $x = 0$ and $x = 3^-$, $F(3-) = 1/2$ and $F(3) = 1$.

(a) Find the probability that X is a non-negative integer, i.e., $P(X \in \{0, 1, 2, 3\})$.

(b) Find $P(2 < X < 2.5)$.

(c) Find $P(1.5 < X \le 3)$.

4. Let Q_n denote the probability that in n tosses of a fair coin, no run of 4 consecutive heads appears.

(a) Find a recursion formula for Q_n in terms of $Q_{n-1}, Q_{n-2}, Q_{n-3}, Q_{n-4}$ (and more if necessary)

(b) Let N be the first n for which a run of 4 consecutive heads appears. Find $E(N)$.

5. The probability of three of a kind in poker is approximately .021128. Use the Poisson approximation to compute the probability that you will get at least two three of a kinds if you play 100 hands of poker.

6. Problem with picture of a df given. I'll give a picture of a df and ask questions such as: the prob. that the rv is between two numbers, the expectation, the variance, etc.

7. Suppose a die is rolled repeatedly and let T_k be the number of rolls until an odd number appears k times. Find a formula for the distribution of T_2 and T_3. Find $E(T_2)$, etc.

8. Let X be a normal random variable with mean 2 and variance 9. Find $P\{X > 5\}$ to four decimal places.

9. A manufacturer is interested in the reliability of the light bulbs it produces. Light bulbs typically fail when they are switched on and not while they are functioning.

   Suppose that the probability of failure each time the light bulb is switched on is p and the performance on a given trial has no effect on the future performance unless the bulb fails, in which case all future trials fail with certainty. The manufacturer is interested in the probability that the light bulb functions at least 100 times.

   For what values of p is this probability at least 90%?

10. Let X be a Poisson random variable. If $P\{X = 1 | X \leq 1\} = 0.8$, what is the mean?

11. A company prices its hurricane insurance using the following assumptions:

    (i) In any calendar year, there can be at most one hurricane.

    (ii) In any calendar year, the probability of a hurricane is 0.05.

    (ii) The occurrence of a hurricane in a calendar year is independent of the occurrence of a hurricane in any other calendar year.

    Using the company's assumptions, calculate the probability that there are fewer than 3 hurricane in a 20-year period.

12. A group insurance policy covers medical claims of the employees of a small company. The value, V, of the claims made in one year is described by $V = 100,000Y$, where Y is a random variable with density function

$$f(y) = \begin{cases} k(1-y)^4, & \text{for } 0 < y < 1, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Find k.

(b) What is the conditional probability that V exceeds 40,000, given that V exceeds 10,000?

13. The value, ν, of an appliance is based on the number of years since purchase, t, as follows:

$$\nu(t) = e^{(7-0.2t)}.$$

If the appliance fails within seven years of purchase, a warranty pays the owner the value of the appliance. After seven years, the warranty pays nothing. The time until failure of the appliance has exponential distribution with mean 10.

Calculate the expected payment from the warranty.

14. Let X equal the number of students who use a library catalog during a 15 minute interval. Assume that X has a Poisson distribution with mean 5. Let W equal the time in minutes between two students arrivals, so that W has an exponential distribution with mean 3. Find

(a) $P\{W > 6\}$, and

(a) $P\{W > 12 | W > 6\}$.

15. To determine the effectiveness of a certain diet in reducing the amount of cholesterol in the bloodstream, 100 people are put on the diet. After they have been on the diet for a sufficient length of time, their cholesterol will be measured. The nutritionist running this experiment has decided to endorse the diet if a least 65 percent of the people have a lower cholesterol level after going on the diet.

    Suppose the new diet has no effect on the cholesterol level.

    (a) Write an expression (in terms of a sum) for the probability that the nutritionist endorses the new diet (do not evaluate the expression).

    (b) Use a normal distribution to approximate the probability that the nutritionist endorses the new diet.

    HINT: If the diet has no effect on the cholesterol, then each person's cholesterol may be higher or lower, strictly by chance. In other words, you can assume that the probability of a person's cholesterol being lower is 1/2.

9. Buses arrive at a specified stop at 15-minute intervals starting at 7am. That is, they arrive a 7:00, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7:00 and 7:30, find the probability that the passenger waits

   (a) less than 5 minutes for a bus;

   (b) more than 10 minutes for a bus.

10. Let X be a continuous random variable with cumulative distribution function $F(x) = x^2$, $0 \leq x \leq 1$.

    (a) Find the probability density function of X.

    (b) Find the probability density function of $Y = X^2$.

    (c) Find $\mathrm{E}(Y^4)$.