1. A 5 card hand is drawn from a standard 52 card deck. Find the probability that the number of spades exceeds the number of hearts by at exactly 3.

2. Assume that a probability mass function is given by:

$$p(x) = \begin{cases} c\frac{1}{x(x+1)} & \text{, if } x = 1, \cdots, n \\ 0 & \text{, otherwise.} \end{cases}$$

   (a) Find c in a closed form expression.

   (b) Find the cumulative distribution function.

3. Recall that $\liminf_i E_i = \cup_{n=1}^{\infty} \cap_{i=n}^{\infty} E_i$ and $\limsup_i E_i = \cap_{n=1}^{\infty} \cup_{i=n}^{\infty} E_i$. Show that $\liminf_i E_i \subseteq \limsup_i E_i$.

   For the next problem there are three urns, I, II, III. In I there are 3 black, 4 white and 5 red balls. In II there are 3 black, 5 blue and 2 red. In III there are 5 black, 4 blue and 11 red.

4. Two urns are chosen at random and then two balls are selected from the chosen urns. Find the probability that 2 blacks and 2 reds are chosen.

5. A fair die is repeatedly tossed until a 2 appears for the 10th time. Write an expression for the probabililty that at least 100 tosses are required. Write the answer in the form $P(\text{Bin}(n, k) \leq k)$.

6. If (X, Y) has joint density

$$f(x, y) = \begin{cases} cxy & \text{, } 0 < x < 2y < 1 \\ 0 & \text{, otherwise.} \end{cases}$$

   (a) Find c.

   (b) Find the $P(X < Y)$.

7. If X (the time of the second accident at a given intersection), has density

$$f(x) = \begin{cases} cxe^{-x} & \text{,} 0 < x < \infty \\ 0 & \text{,otherwise.} \end{cases}$$

   (a) Find c.

   (b) Find $E(1/X)$.

8. Find $P(1 \leq X \leq 1.5)$ for a r.v., X, which has the c.d.f. below:

9. The number of telephone calls coming into the central switchboard of an office building has a Poisson distribution and averages four per minute.

   (a) Find the probability that the first call will occur between 2 and 3 minutes.

   (b) Find the probability that at least 2 calls will arrive in the first 8 minutes.

10. Of the 100 people in a certaain village, 50 always tell the truth, 30 always lie and 20 always refuse to answer. A sample of size 30 is taken with replacement. Find the probability that the sample will contain 10 people of each category.

11. Two coins are available, one fair and the other two-headed. Choose a coin at random and toss it once; assume that the faircoin is chosen with probability 3/4. Given that the result is heads, find the probability that the two-headed coin was chosen.

12. Three fair dice are thrown. Find the probability that the middle number is between 2 and 4 and the other two are not.

13. Person Q arrives at a bus stop at a time which is uniformly distributed between 10am and 11am. The time of arrival of the bus is independent of the time of arrival of Q and is also uniformly distributed between 10am and 11am. Find the probability that Q will get there before the bus.

14. The length of life, L, of a type of automatic washer is exponentially distributed with mean 2. If 100 such washers are tested, what is the probability that the number lasting longer than $2\ln 2$ years is ≥ 40?

15. Three movie theaters compete for $15,000$ customers. Suppose that each customer chooses one of the three with "total indifference" and independently of other customers. How many seats should each theater have so that the probability of turning away any customer for lack of seats is less than .01?

16. Let X and Y be independent with the (same) density given by:

$$f(x) = \begin{cases} 4x^3 e^{-x^4} & \text{, if } x > 0 \\ 0 & \text{, otherwise.} \end{cases}$$

   Find $E(X+Y)^3$.

17. If X has a Pois(2) distribution, show that $P(X \text{is even}) = 1 - \frac{e^{-4}}{2}$.