1. Definitions.
   (a) Vector space.
   (b) Subspace.
   (c) Well-defined.
   (d) Linear transformation
   (e) Null Space or Kernel
   (f) One to one.
   (g) Onto.
   (h) Self-adjoint.
   (i) Range
   (j) Quotient space
   (k) Projection operator
   (l) Isomorphic
   (m) Inner product.
   (n) $M^\perp$.
   (o) $Ann(M)$.
   (p) An operator on an inner product space, $T$, is symmetric ...
   (q) Span
   (r) Linearly independent
   (s) Basis
   (t) Dimension
   (u) Dual space.
   (v) Definition of Adjoint, $T^*$.
   (w) Linear functional
   (x) Norm
   (y) Trace
   (z) Nilpotent.

2. Theorems.

3. The image of a subspace is a subspace.

4. $M + x = M + y$ if and only if $x - y \in M$.

5. Thm. $\text{Hom}(V, V)$ is a vector space.

6. Thm: $T : V_1 \longrightarrow V_2$. $T$ is 1-1 iff $N(T) = 0$.

7. Thm: If $V_1$ is isomorphic to $V_2$, then $\dim(V_1) = \dim(V_2)$.

8. The Isomorphism Theorem.

9. $T$ is 1 – 1 if and only if $\ker(T) = 0$.

10. Corollary. Suppose $T : V \longrightarrow V$. Then $\dim(V) = \dim(R) + \dim(N)$.

11. Corollary. $\dim(V/N) + \dim(N) = \dim(V)$.

12. Cauchy-Schwarz Inequality
13. Theorem: $M^\bot$ is a subspace.


15. Parallelogram Law.

16. If $T : V \rightarrow V$, $V$ is an inner product space and $T$ is self-adjoint (symmetric), then $\ker(T) \cap \mathcal{R}(T) = (0)$.

17. Thm: $V, < , >$, given; $M$ a subspace of $V$, then $V = M + M^\bot$.

18. Theorem: Given $L : V \rightarrow W$ and $T$ a subspace of $W$ and $L^{-1}(T) = v?V$ s.t. $L(v) ? T$, $L^{-1}(T)$ is a subspace of $V$.

19. Theorem: Given a basis $x_i$ for $V$, each point in $v?V$ has a unique representation $v = \sum_{i=1} c_i x_i$.

20. Theorem: All bases of $V$ have the same number of vectors. (Called the dimension of $V$).

21. Theorem: Given an isomorphism $T : V \rightarrow W$ and \{v_i\} a basis for $V$, \{T(v_i)\} is a basis for $W$.

22. Gram-Schmidt procedure.

23. Theorem: Given $T : V \rightarrow V$ and $y?V$ there exists a unique $z?V, < Tx, y >= < x, z >$ for all $x?V$.

24. Theorem: If $f \in V^*$, there $z?V$ s.t. $f(x) = < x, z >$.

25. Theorem: If $V$ has a basis \{b_j\}_{j=1}^n$ and $T : V \rightarrow V$ has the associated $n \times n$ matrix $A$, then the matrix associated with $T^*$ is $A^T$.

26. Theorem: The dimension of the column space of $A$ equals the dimension of the row space of $A$.

27. Theorem: Given $T : V \rightarrow W$, $\dim(\text{Range}(T)) = \dim(\mathcal{R}(T^*))$.

28. Theorem: Given a projection $P : V \rightarrow M, M$ a subspace of $V$, $P^2 = P$.


31. trace($AB$) = trace($BA$).

32. trace($ABC$) = trace($CAB$).

33. trace($A^*$) = trace($A$).

34. Row space = $\mathcal{R}(T^*)$.

35. Column space = $\mathcal{R}(T)$.

36. $\dim(V^*) = \dim(V)$. 

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