1. **Definitions.**
   (a) Vector space.
   (b) Subspace.
   (c) Well-defined.
   (d) Linear transformation
   (e) Null Space or Kernel
   (f) One to one.
   (g) Onto.
   (h) Self-adjoint.
   (i) Range
   (j) Quotient space
   (k) Projection operator
   (l) Isomorphic
   (m) Inner product.
   (n) $M^{\perp}$.
   (o) $Ann(M)$.
   (p) An operator on an inner product space, T, is symmetric ... .
   (q) Span
   (r) Linearly independent
   (s) Basis
   (t) Dimension
   (u) Dual space.
   (v) Definition of Adjoint, T^*.
   (w) Linear functional
   (x) Norm
   (y) Trace
   (z) Nilpotent.
2. **Theorems.**
3. The image of a subspace is a subspace.
4. $M + x = M + y$ if and only if $x - y \in M$.
5. Thm. $\text{Hom}(V, V)$ is a vector space.
6. Thm: $T : V_1 \longrightarrow V_2$. T is 1-1 iff $N(T) = 0$.
7. Thm: If V_1 is isomorphic to V_2, then $\dim(V_1) = \dim(V_2)$.
8. The Isomorphism Theorem.
9. T is $1 - 1$ if and only if $\ker(T) = 0$.
10. Corollary. Suppose $T : V \longrightarrow V$. Then $\dim(V) = \dim(R) + \dim(N)$.
11. Corollary. $\dim(V/N) + \dim(N) = \dim(V)$.
12. Cauchy-Schwarz Inequality

13. Theorem: $M^{\perp}$ is a subspace.
14. Parallelogram Law.
15. Triangle inequality.
16. If $T : V \longrightarrow V, V$ is an inner product space and T is self-adjoint (symmetric), then $\ker(T) \cap \mathcal{R}(T) = (0)$.
17. Thm: $V, < .,. >$, given; M a subspace of V, then $V = M + M^{\perp}$.
18. Theorem: Given $L : V \longrightarrow W$ and T a subspace of W and $L^{-1}(T) = v?V s.t. L(v)?T, L^{-1}(T)$ is a subspace of V.
19. Theorem: Given a basis x_i for V, each point in v?V has a unique representation $v = \sum_{i=1} c_i x_i$
20. Theorem: All bases of V have the same number of vectors. (Called the dimension of V).
21. Theorem: Given an isomorphism $T : V \longrightarrow W$ and $\{v_i\}$ a basis for V, $\{T(v_i)\}$ is a basis for W.
22. Gram-Schmidt procedure.
23. Theorem: Given $T : V \longrightarrow V$ and $y?V$ there exists a unique $z?V, < Tx, y >=< x, z >$ for all $x?V$.
24. Theorem: If $f \in V^*$, there $z?V s.t. f(x) =< x, z >$.
25. Theorem: If V has a basis $\{b_j\}_{j=1}^{n}$ and $T : V \longrightarrow V$ has the associated $n \times n$ matrix A, then the matrix associated with T^* is A^T.
26. Theorem: The dimension of the column space of A equals the dimension of the row space of A.
27. Theorem: Given $T : V \longrightarrow W$, $\dim(Range(T)) = \dim(\mathcal{R}(T^*))$.
28. Theorem: Given a projection $P : V \longrightarrow M, M$ a subspace of V, $P^2 = P$.
29. Theorem: Given $S, T : V \longrightarrow W, (S + T)^* = S^* + T^*$.
30. Theorem: Given $S, T : V \longrightarrow W, (ST)^* = T^* S^*$.
31. $\text{trace}(AB) = \text{trace}(BA)$.
32. $\text{trace}(ABC) = \text{trace}(CAB)$.
33. $\text{trace}(A^*) = \text{trace}(A)$.
34. Row space $= \mathcal{R}(T^*)$.
35. Column space $= \mathcal{R}(T)$.
36. $\dim(V^*) = \dim(V)$.