Math 423 **The real quiz 5** **Name:**__________________________

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1. Define the linear transformation (or matrix) $M : \mathbb{R}^6 \longrightarrow \mathbb{R}^6$ by

$$M = \begin{bmatrix} 1 & 0 & -1 & 1 & -2 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Assume (I'm giving you this information) that

$$\mathcal{R}(M^2) = \text{sp}(e_1 + e_2 + e_3) = \text{sp}(M^2 e_1), Me_1 = e_1 + e_2, \text{etc.}$$

$$\ker(M) = \text{sp}(e_1 + e_4 + e_5, e_2 - e_3 - e_4 - e_5, e_6).$$

   (a) Show that M is nilpotent.

   (b) Find the basis which gives the Jordan form for the matrix, M.

   (c) Find the Jordan matrix, J, which is similar to M.

2. Prove that if $M \subseteq M + N$ and $\beta = \{b_1, b_2, \ldots, b_k\}$ is a basis for M, then there exists a basis for $M + N$, which extends β, say, $\gamma = \{b_1, b_2, \ldots, b_k, c_1, \ldots, c_l\}$ such that the new elements, $\{c_j\}_{j=1}^{l}$, are in N.