Random $\varepsilon$-nets and embeddings in $\ell^N_{\infty}$

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We show that, given an $n$-dimensional normed space $X$, a sequence of $N = (8/\varepsilon)^{2n}$ independent random vectors $(X_i)_{i=1}^N$, uniformly distributed in the unit ball of $X^*$, with high probability forms an $\varepsilon$-net for this unit ball. Thus the random linear map $\Gamma : \mathbb{R}^n \to \mathbb{R}^N$ defined by $\Gamma x = (\langle x, X_i \rangle)_{i=1}^N$ embeds $X$ in $\ell^N_{\infty}$ with at most $(1 + \varepsilon)$-norm distortion. In the case $X = \ell^n_2$ we obtain a random $(1+\varepsilon)$-embedding into $\ell^N_{\infty}$ with asymptotically best possible relation between $N$, $n$, and $\varepsilon$.

This is joint work with Y. Gordon, A. Pajor, and N. Tomczak-Jaegermann.