Invertibility of random matrices

Mark Rudelson*

Joint work with Roman Vershynin.

Let A be an $n \times n$ matrix with independent entries. It is well known that under appropriate moment assumptions, the norm of A is of order $\sqrt{n}$ with high probability. It was long believed that the same holds for A^{-1}. This was only known for Gaussian matrices, and was conjectured for Bernoulli matrices (with independent uniform ± 1 entries). We prove this conjecture in full generality, for matrices with general independent entries, and under mild moment assumptions. Moreover, for any $\varepsilon > e^{-cn}$ we obtain a bound on the norm of A^{-1}, which holds with probability $1 - \varepsilon$, and show that such bound is optimal.

The proof is based on combination of classical methods of asymptotic geometric analysis and delicate anti-concentration inequalities described in the talk of Roman Vershynin.

*Department of Mathematics, University of Missouri, Columbia, MO 65211, USA; e-mail: rudelson@math.missouri.edu