Sums of independent random variables and arithmetic structure I and II

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The classical concentration inequalities of probability theory demonstrate that nice random variables, such as the sums of i.i.d.'s, are nicely concentrated about their means. Less studied but often needed is the reverse phenomenon – that the concentration is not too tight. The Littlewood-Offord Problem asks, for i.i.d. random variables X_k and real numbers a_k, to put an upper bound on the probability P that the sum $\sum a_k X_k$ lies near some number v.

Unlike the concentration inequalities, which are controlled only by the magnitude of the coefficients a_k (e.g. the norms of the coefficient vectors, like in Berstein's inequality), the anti-concentration inequalities are less stable; they are controlled by the arithmetic structure in the coefficients a_k. Tao and Vu put forth a program to show that if the concentration probability P is large then there is a strong additive structure in the coefficients $a_1, ..., a_n$: this sequence should be embedded in a short arithmetic progression (or a generalization thereof). This says that the only reason for a random sum to concentrate too much is due to (essential) cancellations, which can only be caused by a rich additive structure of the summands.

I will describe a solution of the Littlewood-Offord problem for arbitrary coefficients a_k of the same order of magnitude. We show that a_k must essentially lie in an arithmetic progression of length $1/P$.

Mark Rudelson will describe how this result is useful for problems in random matrix theory.

Joint work with Mark Rudelson.