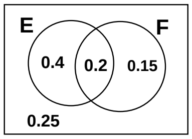
Chapter 7 Homework Solutions

Compiled by Joe Kahlig

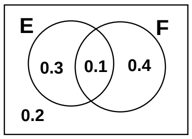
---

1. (a) S={(heads, red), (heads, white), (tails, red), (tails, white)}
   (b) There are multiple answers for this part. Any two disjoint subsets of S are acceptable.
   E = { (h,r), (h,w)}
   F = { (t,w)}
2. (a) Since we are drawing them out simultaneously, we don't care about the order. i.e. (1,2) is the same as (2,1)
   $S = \{(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(2,3),$
   $(2,4),(2,5),(2,6),(2,7),(3,4)(3,5),(3,6),$
   $(3,7),(4,5),(4,6),(4,7),(5,6),(5,7),(6,7)\}$
   (b) $E = \{(1,3),(1,5),(1,7),(3,5),(3,7),(5,7)\}$
   (c) $F = \{(2,4),(2,6),(4,6)\}$
   (d) No.
   (e) There are multiple answers for this part. Any two disjoint subsets of S are acceptable.
   G = {(1,3), (1,4), (1,5), (2,3)}
   H = {(2,5), (3,4), (3,5), (4,5)}
3. (a) S = { 6, 10, 11, 15, 20}
   (b) Not equally likely since the probability of getting 20 cents is $\frac{C(2,2)}{C(6,2)} = \frac{1}{15}$ and the probability of getting 10 cents is $\frac{C(3,2)}{C(6,2)} = \frac{1}{5}$. Since these are not the same, the sample is not equally likely. Note that we used concepts in section 7.4 to compute these probabilities.
4. Answers will vary.
   E = {HHH}
   F = {HHT, HTT, TTT}
5. Let w = white ball, g=green ball, and y = yellow ball.
   (a) Note that order is important.
   S= { ww, wg, wy, gw, gg, gy, yw, yg}
   (b) G = { wg, gw, gy, yg}
   (c) Answers will vary, but pick E such that $E \cap G = \emptyset$
   One answer is: E={ww, wy}.
6. (a) S = {R, E, P, S, N, T, A, I, V }
   (b) $2^{n(S)} = 2^9 = 512$
   (c) E = {E,A, I}
7. (a) No. There are more red balls in the bag, so the drawing a red ball is more likely than drawing a white ball.
   (b) See part (a) for the answer since uniform and equally likely mean the same thing.
8. (a) $0.2 = 1 - (.15 + .25 + .4)$
   (b) $0.4 = .15 + .25$
9. Since $P(a) + P(b) + P(c) = 1$ and $P(a) + P(b) = 0.75$, then $P(c) = 0.25$. Similarly $P(a) = 0.55$ and $P(b) = 0.2$.
10. $J^C = \{d, e\}$ which means that $P(d) + P(e) = 0.4$ and thus $P(d) = 0.25$. Since all probability adds up to 1 we get that $P(c) = 0.2$
11. $J^C = \{a, d, e\}$ which means that
    $P(a) + P(d) + P(e) = 0.45$
    $P(a) + 0.2 + 0.1 = 0.45$
    $P(a) = 0.15$

    Since a and b are equally likely, then $P(b) = 0.15$. Since all probability adds up to 1, we get that $P(c) = 0.4$
12. (a) $\frac{20+7}{90} = \frac{27}{90}$
    (b) $\frac{21}{90}$
13. $\frac{25+30}{210} = \frac{55}{210}$
14. (a) $\frac{6}{11}$
    (b) $\frac{6+2}{11} = \frac{8}{11}$
15. (a) $\frac{41}{713}$
    (b) $\frac{55+41+52}{713} = \frac{181-33}{713} = \frac{148}{713}$
    (c) $\frac{171+199-41}{713} = \frac{329}{713}$
    (d) $\frac{199+141}{713} = \frac{340}{713}$
16. (a) $\frac{85+35}{300}$
    (b) $\frac{85}{300}$
    (c) $\frac{58}{300}$
    (d) $\frac{170+26+154-12-138}{300} = \frac{200}{300}$
17. (a) $\frac{30+20+10+10}{1000} = \frac{70}{1000}$
    (b) $\frac{90+290-30}{1000} = \frac{350}{1000}$
    (c) $\frac{250+320+260}{1000} = \frac{830}{1000}$
18. Use the information to fill in a venn diagram to answer part b and c.

    (a) $0.4 + 0.2 = 0.6$ or $1 - 0.4 = 1 - P(E^C)$
    (b) 0.4
    (c) 0.8
19. Use the information to fill in a venn diagram to answer part c.

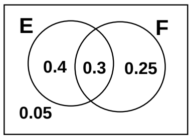
    

(a) $0.4 = 1 - 0.6 = 1 - P(E^C)$

(b) $0.1 = 0.4 + 0.5 - 0.8 = P(E) + P(F) - P(E \cup F)$

(c) 0.3

20. Use the information to fill in a venn diagram to answer part b and c.

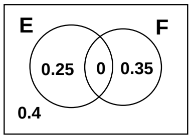


(a) $0.55 = 1 - P(F^C)$

(b) 0.3

(c) $0.3 + 0.25 + 0.05 = 0.6$

21. $P(E \cap F) = 0$, since E and F are mutually exclusive. Use the information to fill in a venn diagram



(a) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$P(E \cup F) = 0.25 + 0.35 - 0 = 0.6$

(b) 0.65

22. (a) $\frac{1}{6} + \frac{1}{8} + \frac{1}{8}$

(b) $\frac{1}{3} + \frac{1}{6}$

(c) $1 - \left(\frac{1}{3} + \frac{1}{6}\right)$

(d) 2^6. An event is the same as a subset.

(e) A and B are mutually exclusive
C and D are mutually exclusive

23. (a) X = a 4 on either die and Y = sum of 5.

Red Die

Green Die	1	2	3	4	5	6
1				XY		
2			Y	X		
3		Y		X		
4	XY	X	X	X	X	X
5				X		
6				X		

Answer: $\frac{2}{36}$

(b) X = a 3 on either die and Y = sum of 4.

Red Die

Green Die	1	2	3	4	5	6
1			XY			
2		Y	X			
3	XY	X	X	X	X	X
4			X			
5			X			
6			X			

Answer: $\frac{12}{36}$

(c) X = a 6 on red die and Y = number less than 3 on the green.

Red Die

Green Die	1	2	3	4	5	6
1	Y	Y	Y	Y	Y	XY
2	Y	Y	Y	Y	Y	XY
3						X
4						X
5						X
6						X

Answer: $\frac{2}{36}$

24. X = a 4 on either die and Y = sum of 7

	1	2	3	4	5	6
1				X		Y
2				X	Y	
3				XY		
4	X	X	XY	X	X	X

Answer: $\frac{11}{24}$

25. Use a venn diagram to organize the information.

Answer: $\frac{180+85}{500} = \frac{265}{500} = 0.53$

26. $\frac{4*3*4*3}{8*7*6*5}$

27. (a) $\frac{8*7*13}{15*14*13}$

(b) $\frac{8*7*3}{15*14*13}$

28. First count the number of ways to hang the posters on the wall so that the posters of the same type are together.The 3! counts the rearrangement of the groups. Divide by the total number of ways to hang the posters on the wall.

Answer: $\frac{(5!4!2!)*3!}{11!}$

29. Choose both Bob and Phill, C(2,2), and choose then 3 of the remaining 18 men. Choose Sara then choose 4 of the remaining 29 women. Divide by the total number of ways to choose 5 men and 5 women. Note: C(2,2) and C(1.1) are both equal to 1 and thus do not have to be included in the answer.

Answer: $\frac{C(2,2)*C(18,3)*C(1,1)*C(29,4)}{C(20,5)*C(30,5)}$

30. Select 3 of the 7 friends and then select 7 of the 93 other applicants.

Answer: $\frac{C(7,3)*C(93,7)}{C(100,10)} = 0.01915$

31. First figure out how many ways the couples may be in the row and then divide by the number of ways 8 people can be placed in a row.

Answer: $\frac{8*1*6*1*4*1*2*1}{8!} = 0.0095$

32. $\frac{C(4,2)+C(5,2)}{C(9,2)}$

33. Use the union formula for counting or probability. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ where A is exactly 4 green marbles and B is exactly 2 blue marbles.

    Answer: $\frac{C(8,4)C(16,2)+C(6,2)C(18,4)-C(8,4)C(6,2)}{C(24,6)}$

34. The 4! is the ordering of the roses on a single shelf. The 8! is the ordering of the flowers on the other two shelves. Multiply by 3 since the roses can be on any of the three shelves.

    Answer: $\frac{3(4!*8!)}{12!}$

35. At least 2 freshmen are the cases: (exactly 2 fr and 1 other) or (exactly 3 fr).

    Answer: $\frac{C(12,2)*C(10,1)+C(12,3)C(10,0)}{C(22,3)} = 0.5714$

36. there are 2 defective and 8 good typewriters.

    (a) $\frac{C(2,0)*C(8,4)}{C(10,4)} = \frac{70}{210} = \frac{1}{3}$

    (b) $\frac{C(2,1)*C(8,3)}{C(10,4)} = \frac{112}{210} = \frac{8}{15}$

37. Select artist A and B and then select two more artist from the remaining 6. Note: $C(2,2) = 1$ so it does not need to be included in the answer.

    Answer: $\frac{C(2,2)*C(6,2)}{C(8,4)}$

38. Select 5 banks from the 6 with discounts and then select 1 bank from the 4 without discounts.

    Answer: $\frac{C(6,5)*C(4,1)}{C(10,6)} = 0.1143$

39. $\frac{C(13,9)C(12,1)+C(13,10)}{C(25,10)}$

40. For this problem it is easier to calculate total - what you don't want. You don't want less than two born in July. If 0 were born in July then there are 11 other months in which people can be born, 11^7 ways. Next if exactly 1 was born in July there are $1 * 11^6$ ways and we multiply this by 7 so that any of the 7 people could be born in July.

    Answer: $1 - \left(\frac{11^7}{12^7} + \frac{7*(1*11^6)}{12^7}\right)$

41. The numerator is a permutation since the day a person is born is acting like a label.

    (a) $\frac{P(365,20)}{365^{20}}$

    (b) easiest way to count this it to do 1 minus what you don't want, which happens to be part (a).

    Answer: $1 - \frac{P(365,20)}{365^{20}} = 0.4114$

42. When picking three people we have the following cases:

Male	Female
0	3
1	2
2	1
3	0

At most two males are the top three cases. notice that we do not want the last case.

Answer: $1 - \frac{C(7,3)}{C(12,3)}$

or $\frac{C(7,2)*C(5,1)+C(7,1)*C(5,2)+C(7,0)*C(5,3)}{C(12,3)}$

43. (a) $P(N|M) = \frac{P(N \cap M)}{P(M)} = \frac{0.25}{.4+.25} = \frac{0.25}{0.65}$

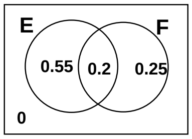
    (b) $P(M|N) = \frac{P(M \cap N)}{P(N)} = \frac{0.25}{.15+.25} = \frac{0.25}{0.4}$

44. (a) $P(J|K) = \frac{P(J \cap K)}{P(K)} = \frac{.3}{.3+.22+.09} = \frac{.3}{.61}$

    (b) $P(M|K^C) = \frac{P(M \cap K^C)}{P(K^C)} = \frac{.14}{.15+.14+.1} = \frac{.14}{.39}$

    (c) $P(M|N) = \frac{P(M \cap N)}{P(N)} = \frac{0}{.09+.1} = 0$

45. Let E = solve the first problem and F = solve the second problem. Fill in a venn diagram with the given information.



    (a) $P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{.2}{.75}$

    (b) $P(E^C|F) = \frac{P(E^C \cap F)}{P(F)} = \frac{.25}{.45}$

46. First organize the information into a table.

	Fresh.(F)	Soph.(S)	Total
Male(M)	6	18	24
Female(F)	1	17	28
Total	7	36	42

Answer: $P(S|M) = \frac{18}{24}$

47. (a) $P(O|\text{only rifle}) = \frac{5}{26}$

    (b) $P(O|\text{own handgun}) = \frac{58+25}{120} = \frac{83}{120}$

    (c) $P(F|\text{own rifle}) = \frac{12+5}{26+35} = \frac{17}{61}$

48. (a) $P(\text{2cds}|\text{over 25}) = \frac{40}{210}$

    (b) $P(19 - -25|\text{lessthan2cds}) = \frac{70+110}{570} = \frac{180}{570}$

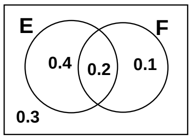
49. We already know two juniors are attending, so we need to determine the remaining three students. We need exactly 1 junior of the remaining 10 and 2 of the 7 remaining students.

    Answer: $\frac{C(10,1)*C(7,2)}{C(17,3)}$

50. We already know three questions: 2 difficult and 1 easy, so we need to determine the remaining questions. We need exactly 2 difficult questions of the remaining 6 and 5 easy questions of the remaining 11.

    Answer: $\frac{C(6,2)*C(11,5)}{C(17,7)}$

51. Use a venn diagram to organize the information.



   (a) $\frac{P(F^C\cap E)}{P(E)} = \frac{0.4}{0.6}$

   (b) $\frac{P(E^C\cap F^C)}{P(F^C)} = \frac{0.3}{0.7}$.

   (c) $\frac{P(F\cap E^C)}{P(E^C)} = \frac{0.1}{0.4}$

52. (a) $P(A^C|C) = \frac{P(A^C\cap C)}{P(C)}$
   $A^C \cap C = \{s_3, s_5\}$
   Answer: $\frac{1/3+1/6}{1/8+1/3+1/6}$

   (b) $P(C|B) = \frac{P(C\cap B)}{P(B)}$
   $C \cap B = \{s_3, s_5\}$
   Answer: $\frac{1/3+1/6}{1/3+1/6+1/12}$

53. (a) $0.6 * 0.3 + 0.4 * 0.2 = 0.26$

   (b) $0.4 * 0.2 + 0.4 * 0.5 = 0.28$

   (c) $0.6 * 0.7 + 0.4 * 0.3 = 0.54$

   (d) $0.4 * 0.5 = 0.2$

   (e) $P(A \cup G) = P(A) + P(G) - P(A \cap G)$
   $P(A \cup G) = 0.6 + 0.54 - 0.42 = 0.72$

   (f) 0.5

   (g) 0.7

   (h) $P(A|G) = \frac{P(A\cap G)}{P(G)} = \frac{0.6*0.7}{0.6*0.7+0.4*0.3}$

   (i) $P(B|R) = \frac{B\cap R)}{P(R)} = \frac{0.4*0.5}{0.4*0.5} = 1$

   (j) $P(A|Y) = \frac{A\cap Y)}{P(Y)} = \frac{0.6*0.3}{0.6*0.3+0.4*0.2}$

   (k) $P(A|R) = \frac{A\cap R)}{P(R)} = \frac{0}{0.4*0.5} = 0$

54. (a) $0.1 * 0.2 + 0.6 * 0.7 = 0.44$

   (b) $0.3 * 0.25 = 0.075$

   (c) 0.8

   (d) $P(C|G) = \frac{C\cap G)}{P(G)} = \frac{0.3*0.75}{0.6*00.3+00.3*0.75}$

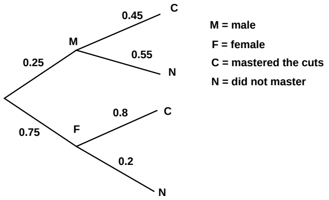
   (e) $P(C) = 0.3$
   $P(E) = 0.1 * 0.2 + 0.6 * 0.7 = 0.44$
   $P(E \cap C) = 0$
   Since $P(E \cap C) \neq P(E) * P(C)$ they are not independent.

   (f) Yes since $P(E \cap C) = 0$

   (g) $P(B) = 0.6$
   $P(E) = 0.1 * 0.2 + 0.6 * 0.7 = 0.44$
   $P(E \cap B) = 0.6 * 0.7 = 0.42$
   $P(E) * P(B) = 0.6 * 0.44 = 0.264$
   Since $P(E \cap B) \neq P(E) * P(B)$ they are not independent.

   (h) No since $P(E \cap B) \neq 0$

55. Draw a tree.



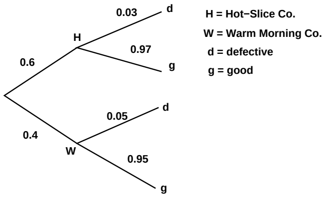
   (a) $P(M|C) = \frac{P(M\cap C)}{P(C)} = \frac{0.25*0.45}{0.25*0.45+0.75*0.8}$

   (b) $P(F \cup C) = .75 + .25 * .45 = 0.8625$

56. The third child has a good squirt gun so there are only 59 good guns remaining. Thus the second child culd pick any of the 20 bad squirt guns out of the total of 59+20=79 squirt guns.

   Answer: $\frac{20}{79}$

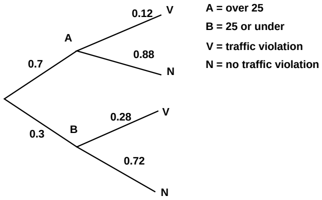
57. Draw a tree.



   $P(d) = 0.6 * 0.03 + 0.4 * 0.05 = .962$

   Answer: 96.2%

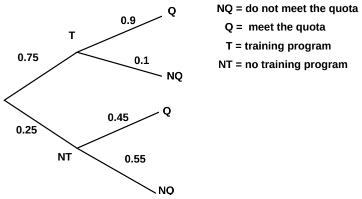
58. Draw a tree.



   (a) $P(A|V) = \frac{P(A\cap V)}{P(V)} = \frac{0.7*0.12}{0.7*0.12+0.3*0.28}$

   (b) $P(V) = 0.7 * 0.12 + 0.3 * 0.28 = 0.168$

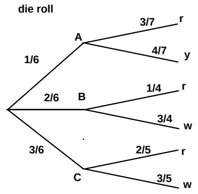
59. Draw a tree.



   (a) $P(T|Q) = \frac{.75*.9}{.75*.9+.25*.45}$

   (b) $P(NQ \cap NT) = .25 * .55$

60. Draw a tree.



(a) $P(C \cap W) = \frac{3}{6} * \frac{3}{5}$

(b) $P(B|r) = \frac{2/6*1/4}{1/6*3/7+2/6*1/4+3/6*2/5}$

61. A club and a diamond have been accounted for so there are still 13 hearts remaining and a total of 50 cards remaining.

Answer: $\frac{13}{50}$

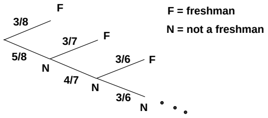
62. (a) $\frac{12}{46}$

(b) $\frac{3}{46}$

(c) The seventh card was the king of hearts.

Answer: 0

63. Think of a tree.



you want $P(N \cap F)$.

Answer: $\frac{5}{8} * \frac{3}{7} = \frac{15}{56}$

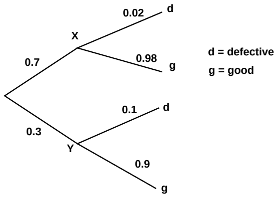
64. Draw a tree similar to the one from problem 63

(a) $\frac{4}{9} * \frac{3}{8} * \frac{5}{7} = \frac{60}{504}$

(b) By the fifth draw you have to have drawn a green ball. since you stop when you draw a green ball, you will never have a sixth draw.

Answer: 0

65. Draw a tree.

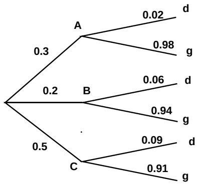


(a) $P(g) = 0.7 * 0.98 + 0.3 * 0.9 = 0.956$

(b) $P(Y|d) = \frac{0.3*0.1}{0.7*0.02+0.3*0.1}$

(c) $P(d \cap Y) = 0.1 * 0.3$

66. Draw a tree.



(a) $P(g \cap (B \cup C))P(g \cap B) + P(g \cap C) =$ $0.2 * 0.94 + 0.5 * 0.91 = 0.643$

(b) $P(g|C) = \frac{P(g \cap C)}{P(C)} = \frac{0.5*0.91}{0.5} = 0.91$

(c) $P(A|d) = \frac{P(A \cap d)}{P(d)} =$ $\frac{0.3*0.2}{0.3*0.02+0.2*0.06+0.5*0.09} = 0.095238$

67. (a) X = 3 or 4 on six sided die
Y = sum greater than 5.

	1	2	3	4	5	6
1			X	X	Y	Y
2			X	XY	Y	Y
3			XY	XY	Y	Y
4		Y	XY	XY	Y	Y

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{5/24}{14/24}$

Answer: $\frac{5}{14}$

(b) X= odd sum greater than 6
Y = 4 on either die

	1	2	3	4	5	6
1				Y		X
2				Y	X	
3				XY		X
4	Y	Y	XY	Y	XY	Y

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{3/24}{9/24}$

Answer: $\frac{3}{9}$

(c) X= sum of 4
Y = sum at most 6

	1	2	3	4	5	6
1	Y	Y	XY	Y	Y	
2	Y	XY	Y	Y		
3	XY	Y	Y			
4	Y	Y				

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{3/24}{14/24}$

Answer: $\frac{3}{14}$

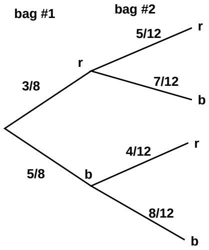
(d) X= sum of 4
Y = roll was a double

	1	2	3	4	5	6
1	Y		X			
2		XY				
3	X		Y			
4				Y		

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{1/24}{4/24}$

Answer:$\frac{1}{4}$

68. (a) probability tree.



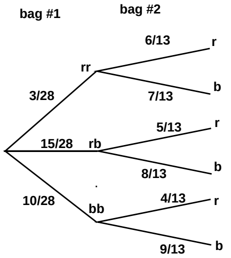
(b) $\frac{3}{8} * \frac{8}{12} + \frac{5}{8} * \frac{8}{12} = \frac{61}{96}$.

(c) P(2^{nd} r | 1^{st} b) = $\frac{4}{12}$

(d) P(1^{st} r | 2^{nd} b) = $\frac{21}{61}$

(e) P(1^{st} r | 2^{nd} 4) = $\frac{3}{7}$

69. (a) The probability of the first level of the tree was computed using combinations and then converting the answers to fractions.



(b) $\frac{15}{28} * \frac{8}{13} + \frac{10}{28} * \frac{4}{13} = \frac{40}{91}$

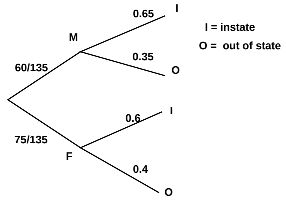
(c) $P(rr \cap b) = \frac{3}{28} * \frac{7}{13} = \frac{3}{52}$

(d) $P(r|rb) = \frac{5}{13}$

(e) $P(rb|r) = \frac{75}{133}$

(f) $P((rr \cup rb)|r) =$
$\frac{(3/28)*(6/13)+(15/28)*(5/13)}{(3/28)*(6/13)+(15/28)*(5/13)+(10/28)*(4/13)} = \frac{93}{133}$

70. Draw a tree.

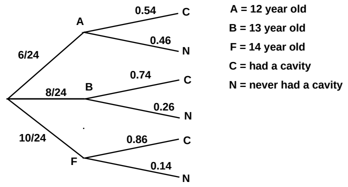


(a) $P(I|M) = 0.65$

(b) $P(F|O) = \frac{.4*70/135}{.35*60/135+.4*75/135} = \frac{10}{17}$

(c) $P(F) = \frac{75}{135} = \frac{5}{9}$
$P(O) = \frac{17}{45}$
$P(F \cap O) = \frac{2}{9}$
Since $P(F)P(O) = \frac{17}{81}$ is not equal to $P(F \cap O)$ these events are dependent. (i.e. not independent)

71. Draw a tree.



(a) $P(F|N) = \frac{.14*10/24}{.46*6/24+.26*8+.14*10/24} = \frac{35}{156}$

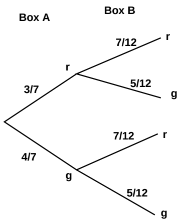
(b) $P(B) = \frac{8}{24}$
$P(C) = \frac{6}{24} * 0.54 + \frac{8}{24} * 0.74 + \frac{10}{24} * 0.86 = 0.74$
$P(B \cap C) = \frac{8}{24} * 0.74 = \frac{37}{150}$
$P(B) * P(C) = \frac{8}{24} * 0.74 = \frac{37}{150}$

Yes, since $P(B \cap C) = P(B) * P(C)$.

72. (a) Since E and F are independent then
$P(E \cap F) = P(E) * P(F)$
$P(E \cap F) = 0.6 * 0.3 = 0.18$

(b) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$P(E \cup F) = 0.6 + 0.3 - 0.18$
Answer: 0.72

73. Since you are drawing an item from each box you can draw this tree to represent the problem.



Answer: $\frac{3}{7} * \frac{7}{13} + \frac{4}{7} * \frac{5}{13}$

74. Note the machines working or not working are independent.

(a) (A breaks down)*(B works all day) + (A works all day)*(B breaks down)
Answer: $0.02 * 0.97 + 0.98 * 0.03$

(b) (A works all day)*(B works all day)
Answer: $0.98 * 0.97$

75. $P(E) = \frac{2}{4}$
$P(F) = \frac{2}{4}$
$P(E \cap F) = \frac{1}{4}$
Since $P(E) * P(F) = \frac{2}{4} * \frac{2}{4} = \frac{1}{4} = P(E \cap F)$, these events are independent.

76. Similar to problem 75
Answer: Independent.

77. Similar to Problem 73
$0.075 * 0.87 + 0.925 * 0.13$

78. (a) $\frac{9}{10} * \frac{17}{20} * \frac{7}{15}$

(b) $\frac{1}{10} * \frac{17}{20} * \frac{7}{15} + \frac{9}{10} * \frac{3}{20} * \frac{7}{15} + \frac{9}{10} * \frac{17}{20} * \frac{8}{15}$