

Week in Review–Additional Chapter 2 Material

Section 2.2 and 2.3: Systems of linear equations.

- Gauss-Jordan row operations.
- Row-Reduced form (Reduced Row Echelon Form)
 - The first non zero number in a row is a one. Called a leading one.
 - The leading one is the only non-zero entry in its column.
 - The leading ones are positioned in a diagonal-like manner starting at the upper left going to the lower right.
- Types of solutions
 - no solutions when a row of the matrix gives a non-true statement, i.e. $0 = 5$.
 - exact solution (when in row-reduced form)
 - number of leading ones = number of variables
 - infinite solution (when in row-reduced form)
 - number of leading ones < number of variables

1. Which of these matrices are in row-reduced form (reduced row echelon form)?

$$(a) \left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{array} \right] \quad (b) \left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$(c) \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 7 \\ 0 & 0 & 1 & 4 & 3 \end{array} \right] \quad (d) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

2. Give the solutions to the system of equations represented by the augmented matrices.

$$(a) \left[\begin{array}{ccc|c} x & y & z & 21 \\ 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 4 \end{array} \right] \quad (b) \left[\begin{array}{ccc|c} x & y & z & 18 \\ 2 & 0 & 0 & 10 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 5 & 0 \end{array} \right] \quad (c) \left[\begin{array}{ccc|c} x & y & z & 2 \\ 1 & 0 & 4 & 9 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$(d) \left[\begin{array}{cccc|c} x & y & z & w & 7 \\ 1 & 2 & 0 & 2 & 3 \\ 0 & 0 & 1 & 4 & 0 \end{array} \right] \quad (e) \left[\begin{array}{ccc|c} x & y & z & 4 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

3. Solve these system of equations with the Gauss-Jordan Method.

- (a) $x + y - 2z = -2$
 $y + 5z = 32$
 $3x + 4y + z = 34$
- (b) $2x - 4y + 6z = 20$
 $3x - 6y + z = 22$
 $-2x + 5y - 2z = -18$

4. solve these systems of equations by any method.

- (a) $3x + y = 9$
 $x + z = 4 + y$
 $z - 11 = -3x$
 $4x + 6y + 2z = 15 + 7y$
- (b) $x + 3y + z = 10$
 $2x + 7y - z = 21$
 $4x + 13y + z = 41$
- (c) $3x + 2y + 5z = 7$
 $x + 4y + z = 13$
 $4x - 5y + 2z = -9$
 $5x + 10y + 7z = 32$

5. Bob has budgeted \$840 to expand his video library with 60 DVDs. His local video store prices their DVDs at \$10 for an old release, \$16 for a semi-new release, and \$22 for a current release. How many DVDs from each category will Bob be able to buy assuming that he uses all of the budgeted money?

(a) Set up and solve the word problem. If the solution is infinite, then place restrictions on the parameter.

(b) How many different solutions are there for this word problem?

Section 2.7: Leontief Input-Output Model

- Formula $X = (I - A)^{-1}D$
 - X is the production matrix. This is the production level of the different sectors.
 - A is the input/output matrix. This is the relationship between what is needed to produce one unit of each sector.
 - D is the consumer demand matrix. This gives the amount of each sector demanded by consumers
 - AX is the matrix that gives the internal consumption of the economy.
6. A village has two industries: farming and woodcutting. They have found that to produce one unit of food it takes 0.3 units of food and 0.1 units of wood. To produce one unit of wood it take 0.5 units of food and 0.1 units of wood. All additional food and wood is exported to a nearby city . The city has a demand of 4350 units of food and 6728 units of wood.
- (a) Give the input-output matrix.
- (b) What level of production is needed by the village to meet the demand of the city?
- (c) How much of the production is used internally by the village?

7. A village has three industries: farming, weaving, and pottery. They have found that to produce one unit of food it takes 0.3 units of food, 0.2 units of cloth and 0.1 units of pottery. To produce one unit of cloth it takes 0.2 units of food, 0.1 unit of cloth, and 0.2 units of pottery. To produce 1 unit of pottery it takes 0.2 units of food, 0.3 units of cloth, and 0.3 units of pottery.
- (a) Give the input-output matrix.
 - (b) If the production level of the village is 40 units of food, 42 units of cloth and 28 units of pottery, how much can of the production will the village be able to export to a nearby city?
 - (c) What production level by the village is needed if the demand in the nearby city is 1356 units of cloth, 678 units of pottery, and 2034 units of food,?
 - (d) For a production level of 90 units of pottery, 150 units of cloth and 120 units of food, how much of the production is used by the village?