

Week in Review—Additional Chapter 2 Material

1. (a) no since there is a 3 in row 1 column 3 position.
 (b) yes
 (c) yes
 (d) yes

2. (a) no solution
 (b) $x = 9$, $y = 10$, and $z = 6$
 (c) $x = 2 - 4z$
 $y = 9 - 5z$
 $z = \text{any number}$
 (d) $x = 7 - 2y - 2w$
 $z = 3 - 4w$
 $y = \text{any number}$
 $w = \text{any number}$
 (e) $x = 4$, $y = 2$, and $z = 8$

3. See the streaming video for the work.

- (a) $x = -6$, $y = 12$, $z = 4$
 (b) $x = 3$, $y = -2$, $z = 1$

4. (a) first rewrite the equations as shown.

$$\begin{aligned} 3x + y &= 9 \\ x - y + z &= 4 \\ 3x + z &= 11 \\ 4x - y + 2z &= 15 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & 0 & 9 \\ 1 & -1 & 1 & 4 \\ 3 & 0 & 1 & 11 \\ 4 & -1 & 2 & 15 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Answer: $x = 2$, $y = 3$, and $z = 5$

(b)
$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 10 \\ 2 & 7 & -1 & 21 \\ 4 & 13 & 1 & 41 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 10 & 7 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Answer:

$$\begin{aligned} x &= 7 - 10z \\ y &= 1 + 3z \\ z &= \text{any number.} \end{aligned}$$

note: no restrictions can be placed on the parameter since this was not a word problem.

(c)
$$\left[\begin{array}{ccc|c} 3 & 2 & 5 & 7 \\ 1 & 4 & 1 & 13 \\ 4 & -5 & 2 & -9 \\ 5 & 10 & 7 & 32 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

Answer: no solution.

5. (a) Set up of the problem:

x = the number of old dvds bought
 y = the number of semi-new dvds bought
 z = the number of new dvds bought.

$$\begin{aligned} x + y + z &= 60 \\ 10x + 16y + 22z &= 840 \end{aligned}$$

Solution:

$$\begin{aligned} x &= 20 + z \\ y &= 40 - 2z \\ z &= \text{any number} \end{aligned}$$

Now place restrictions on the parameter z . This is the mathematical process. You could also do this by inspecting the parametric solution for what values of z will make sense.

We know that the number of dvds bought must be greater than or equal to zero.

$$\begin{aligned} x &\geq 0 & y &\geq 0 & z &\geq 0 \\ 20 + z &\geq 0 & 40 - 2z &\geq 0 & & \\ z &\geq -20 & 40 &\geq 2z & & \\ & & 20 &\geq z & & \end{aligned}$$

We also know that the number of dvds bought must be less than 60.

$$\begin{aligned} x &\leq 60 & y &\leq 60 & z &\leq 60 \\ 20 + z &\leq 60 & 40 - 2z &\leq 60 & & \\ z &\leq 40 & -2z &\leq 20 & & \\ & & z &\geq -10 & & \end{aligned}$$

Thus we get that $0 \leq z \leq 20$ and z must be an integer or in other words $z = 0, 1, 2, 3, \dots, 20$

- (b) 21 different solutions.

6. (a)
$$A = \begin{bmatrix} 0.3 & 0.5 \\ 0.1 & 0.1 \end{bmatrix}$$

- (b) 12550 units of food
8870 units of wood

- (c) 8200 units of food
2142 units of wood

$$7. \quad (a) \quad A = \begin{bmatrix} 0.3 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.3 \\ 0.1 & 0.2 & 0.3 \end{bmatrix}$$

(b) 14 units of food

21.4 units of cloth

7.2 units of pottery

(c) 4620 units of food

3400 units of cloth

2600 units of pottery

(d) 84 units of food

66 units of cloth

69 units of pottery