Week in Review # 2

Section 2.1 Systems of linear equations.

- Intersection of two lines.
  - one solution, no solution, infinitely many solutions.
- Setting up word problems.
  - Define the variables.

1. Solve the system of equations.

   $\frac{5x}{4} + \frac{y}{4} = 1$
   $\frac{x}{3} - \frac{y}{2} = \frac{5}{6}$

2. For what value(s) of k will the system of equations have exactly one solution?

   $-2x + 3y = 9$
   $kx - 2y = -6$

3. Set up the system of equations. Do not solve.

   Seven hundred tickets were sold for a performance of a play. The tickets cost \$6 for seniors, \$8 for adults, and \$3.5 for children. The total receipt for the performance was \$3,512.50. If three times as many children as adults attend the play, how many of each ticket were sold?

4. Set up the system of equations. Do not solve.

   An airline is considering the purchase of eleven aircraft to meet an estimated demand for 3200 seats. They have \$1540 million to spend on three types of planes: Boeing 747s, Boeing 777s and Airbus A321s. The table below shows number of seats and cost(in millions) of each plane. How many of each plane should the airline order to meet its demand for seats?

	Seats	Cost
Boeing 747	400	200
Boeing 777	300	160
Airbus A321	200	60

5. Set up the system of equations. Do not solve.

   Bob has \$82,000 that he wants to invest in the stock market and in bonds. Due to the volatility of the stock market, he has divided the stocks into two categories: low-risk stocks and high-risk stocks. Bob has decided that the amount invested in high-risk stocks should equal the amount invested in low-risk stocks and bonds combined. Bob estimates that the bonds will have a dividend rate of 4%/year; low-risk stocks will have a rate of return of 8%/year; and high-risk stocks will have a rate of return of 15%/year. How much money should Bob invested in each category, if he wants to have a return of \$9050/year on the total investment?

Section 2.4: Matrices

- dimension(size) row x columns
  - a_{ij} is the entry in row i and column j
- scalar product of a matrix

- transpose of a matrix, denoted A^T
- addition/subtraction of matrices
  - must be the same dimension.
- if the matrix has a variable in it, then you must do the computation by hand.

$$A = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 9 & 3 & 0 \\ -1 & 2 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 3 \\ -2 & 5 \\ 2 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} \text{x} & 1 \\ 2 & 5 \end{bmatrix}$$

6. Compute the following with the matrices listed above.

   (a) $3d_{2,2} + 2c_{2,1} =$

   (b) $3A =$

   (c) $C^T =$

   (d) $A + 2B =$

   (e) $3A - 4B =$

   (f) $6A - 2B + 3C =$

   (g) $5C - 2A^T =$

7. Solve for x and y.

$$4\begin{bmatrix} 6 & 2\text{x} \\ \text{y} & 5 \end{bmatrix} - \begin{bmatrix} 5 & 3\text{y} \\ 18 & 10 \end{bmatrix} = \begin{bmatrix} 19 & \text{x} \\ -28 & 10 \end{bmatrix}^T$$

Section 2.5: Matrix Multiplication

- if A is a mxn matrix and B is a nxp matrix then the matrix AB has a size of mxp
  - note: the number of columns of A equals the number of rows of B.
- identity matrix I_n
  - size nxn
  - all zeros except for $a_{1,1} = a_{2,2} = a_{3,3} = ... = 1$

$$A = \begin{bmatrix} 7 & 2 & 4 \\ 6 & 5 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 9 & 3 & 0 \\ -1 & 2 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 3 \\ -2 & 5 \\ 2 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} \text{x} & 1 \\ 2 & 5 \end{bmatrix}$$

8. Use the above matrices to compute the following.

   (a) $AC =$

   (b) $BD =$

   (c) $DB =$

   (d) $D^2 =$

   (e) $2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 8 \\ 2 & 3 & 0 \\ 0 & 1 & 5 \end{bmatrix} =$

9. A dietitian plans a meal around two foods. The number of units of vitamin A and vitamin C in each ounce of these foods is represented by the matrix M.

$$M = \begin{array}{c} \\ \text{Vitamin A} \\ \text{Vitamin C} \end{array} \begin{array}{c} \begin{array}{cc} \text{Food I} & \text{Food II} \end{array} \\ \left[\begin{array}{cc} 30 & 90 \\ 7 & 45 \end{array}\right] \end{array} \qquad B = \begin{array}{c} \begin{array}{cc} \text{Food I} & \text{Food II} \end{array} \\ \left[\begin{array}{cc} 5 & 2 \end{array}\right] \end{array} \qquad L = \begin{array}{c} \begin{array}{cc} \text{Food I} & \text{Food II} \end{array} \\ \left[\begin{array}{cc} 9 & 4 \end{array}\right] \end{array}$$

The matrices B and L represent the amount of each food (in ounces) consumed by the girl at breakfast and lunch, respectively. Explain the meaning of the entries in these computations..

(a) $LM = \begin{bmatrix} 298 & 990 \end{bmatrix}$

(b) $MB^T = \begin{bmatrix} 330 \\ 125 \end{bmatrix}$

Section 2.6: The inverse of a Matrix

- the matrix must be square.
- <u>NOT</u> all square matrices have an inverse.
- the inverse of A is denoted A^{-1}
- $AA^{-1} = A^{-1}A = I$
- A system of equations may be written as a matrix equation: AX=B
  - A is the coefficient matrix
  - X is the variable matrix
  - If A has an inverse, then the solution is $X = A^{-1}B$.
    - Matrix A not having an inverse does not imply that the system of equations has no solution. It means that you need to try another method to solve the problem.

10. If $A = \begin{bmatrix} 5 & 1 \\ 5 & 2 \end{bmatrix}$ find A^{-1}

11. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ find A^{-1}

12. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ find A^{-1}

13. Answer the following using this system of equations. $\begin{array}{rcl} 2x \quad\; + z & = & 2 \\ 2x + y - z & = & 1 \\ 3x + y - z & = & 4 \end{array}$

    (a) Write down the coefficient matrix.

    (b) Write the system of equations as a matrix equation.

    (c) Solve the system of equations using matrices.

14. Solve for the matrix X. Assume that all matrices are square and all needed inverses are possible.

    (a) $BX = E - CX$

    (b) $XJ + XA = K$