INSTRUCTIONS

1. In Part I (Problems 1-19) circle the correct choice on your exam. Use the back of each page for scratch work. This part will be collected after 90 minutes; calculators will not be allowed during that hour.

2. In Part II (Problems 20-27), write all solutions in the space provided. Calculators are not allowed. CLEARLY INDICATE YOUR FINAL ANSWER.
Part I - Multiple Choice

For problems 1 through 19, circle the correct answer on your exam. Each question is worth 4 points.

1. Use differentials to approximate $\sqrt{11}$.
   a) $\frac{8}{3}$  
b) $\frac{11}{3}$  
c) $\frac{23}{6}$  
d) $\frac{21}{6}$  
e) $\frac{10}{3}$

2. If $f(x) = 3x \cos^2(x^2)$, find $f'(0)$.
   a) 0  
b) $-3$  
c) 3  
d) 1  
e) $-9$

3. Compute $\log_4 2$
   a) 4  
b) $\frac{3}{2}$  
c) $\frac{1}{2}$  
d) 2  
e) $\frac{2}{3}$

4. Two sides of a triangle are fixed at 4cm and 6cm and the angle between them is increasing at a rate of .02 radians per second. How fast is the area of the triangle increasing when the angle between them is $\frac{\pi}{6}$?
   a) $(.12)\sqrt{3}$  
b) $\frac{.02}{6}$  
c) $\frac{.02}{6\sqrt{3}}$  
d) .12  
e) $12 \sin(.02)$
5. Let \( f(x) = (1 + x^2)^{\frac{3}{2}} \). Then \( f''(0) = \)

a) 3  

b) 0  

c) 6  

d) \( \frac{3}{4\sqrt{2}} \)  

e) \( \frac{3}{4} \)  

6. Solve for \( x \): \( \log(3 - x) + \log(x + 4) = 1 \)

a) \( x = \frac{-1 \pm \sqrt{89}}{2} \)  

b) \( x = 1 \) only  

c) no solution  

d) \( x = \frac{-1 + \sqrt{89}}{2} \) only  

e) \( x = 1 \) or \( x = -2 \)  

7. The function \( f(x) = x^3 + 5x - 1 \) is one-to-one. Let \( g = f^{-1} \). Then \( g'(5) = \)

a) 8  

b) \( \frac{1}{80} \)  

c) \( \frac{8}{25} \)  

d) \( \frac{1}{8} \)  

e) 80  

8. Given the curve parametrized by \( x = t^3 - 3t^2 - 9t + 1 \), \( y = t^3 + 3t^2 - 9t + 1 \), at which point does the curve have a vertical tangent?

a) \( (1, -3) \)  

b) \( (6, 12) \)  

c) \( (-10, 6) \)  

d) \( (-1, 3) \)  

e) \( (1, 1) \)
9. \( \lim_{{x \to 0}} \frac{4 \cos x - 4 + 3 \sin x}{5x} = \)

a) \( \frac{4}{5} \) \quad b) \( -\frac{4}{5} \) \quad c) \( \frac{3}{5} \)

d) 1 \quad e) 0

10. Find the slope of the line tangent to the curve given by \( y^2 + xy = 8 \) at the point \((-2, -2)\).

a) \(-2\) \quad b) \(-\frac{10}{3}\) \quad c) \(-\frac{1}{3}\)

d) \(-3\) \quad e) 0

11. Which of the following statements is true about the curve \((2 + \cos t)i + (1 + \sin t)j\)?

a) Clockwise movement around the circle \((x - 2)^2 + (y - 1)^2 = 1\)

b) Counterclockwise movement around the circle \((x - 2)^2 + (y - 1)^2 = 1\)

c) Clockwise movement around the ellipse \(x^2/4 + y^2 = 1\)

d) Counterclockwise movement around the ellipse \(x^2/4 + y^2 = 1\)

e) None of the above statements is correct.

12. Let \( f(x) \) be a differentiable function and let \( g(x) = 3x^2 - 1 \). Let \( H(x) = f(g(x)) \), the composite of \( f \) and \( g \). If \( f(0) = 1, f'(0) = -1, f(1) = 3, f'(1) = 2, f(2) = -1, f'(2) = 5 \), find \( H'(1) \).

a) 30
b) 12
c) -6
d) 6
e) 5

13. What is the domain of \( \ln(x^2 - 4) \)?

a) \(|x| \geq 2\)
b) \(|x| > 2\)
c) \(|x| \leq 2\)
d) \(|x| < 2\)
e) \(x > 0\)
14. \( \lim_{x \to \infty} 3^{1-x} = \)

a) 0  
b) \( \infty \)  
c) \(-\infty\)  
d) 1  
e) 3

15. Find the domain and range of the inverse of \( f(x) = \frac{3x - 5}{7x + 2} \)

a) Domain: All real numbers except \( \frac{2}{7} \); Range: All real numbers except \( -\frac{3}{7} \)

b) Domain: All real numbers except \( -\frac{2}{7} \); Range: All real numbers except \( \frac{3}{7} \)

c) Domain: All real numbers except \( \frac{3}{7} \); Range: All real numbers except \( -\frac{2}{7} \)

d) Domain: All real numbers except \( \frac{5}{3} \); Range: All real numbers.

e) None of the above is correct.

16. If \( \langle \cos 3t, t \rangle \) is the position of an object at time \( t \), find the acceleration of the object at time \( t = \frac{\pi}{9} \).

a) \( \langle \frac{1}{2}, 0 \rangle \)

b) \( \langle -\frac{1}{2}, 0 \rangle \)

c) \( \langle -\frac{9}{2}, 0 \rangle \)

d) \( \langle \frac{9}{2}, 0 \rangle \)

e) \( \langle 3, 0 \rangle \)
17. If \( f(x) = e^{x\tan x} \), find \( f'(x) \).

a) \( f'(x) = e^{x\tan x} \)

b) \( f'(x) = \sec^2 xe^{x\tan x} \)

c) \( f'(x) = (\tan x + x \sec^2 x)e^{x\tan x} \)

d) \( f'(x) = (\tan x + x \sec x \tan x)e^{x\tan x} \)

e) \( f'(x) = x \tan xe^{x\tan x-1} \)

18. Find the equation of the tangent line to the graph of \( x = e^{2t}, y = te^t \) at the point \((1, 0)\).

a) \( y = 2x - 1 \)

b) \( y = 4x - 4 \)

c) \( y = \frac{1}{2}x - \frac{1}{2} \)

d) \( y = \frac{1}{3}x - \frac{1}{3} \)

e) \( y = x - 1 \)

19. Find the quadratic approximation for \( f(x) = \frac{1}{x} \) at \( x = 1 \).

a) \( x^2 - 3x + 3 \)

b) \( x^2 - x + 2 \)

c) \( x^2 - 2x + 1 \)

d) \( x^2 + 4x + 5 \)

e) \( x^2 + x - 3 \)
**Part II - Work Out Problems**

Work the following problems in the space provided. No calculators

20. a.) Find the linear approximation for \( f(x) = \sqrt[4]{x + 1} \) at \( x = 0 \).
    
b.) Use part a.) to obtain an approximation to \( \sqrt[4]{1.01} \).

21. The position of a particle is given by \( \mathbf{r}(t) = \left\langle \frac{\cos t}{e^t}, \frac{\sin t}{e^t} \right\rangle \). Find the velocity and speed of the particle when \( t = 0 \).

22. The radius of a sphere was given to be 8 inches with a maximum possible error in measurement of 0.01 inches. Use differentials to estimate the maximum error in the calculated volume of the sphere.

23. Find all values of \( x \) between 0 and \( 2\pi \) where the tangent line to \( f(x) = 2x - \tan x \) is horizontal.

24. A trough is 20 feet long. The end of the trough is an isosceles triangle with height 10 feet and length of 3 feet across the top. If water is poured in the trough at a rate of 3 cubic feet per minute, how fast is the water level rising when the height of the water is 1 foot?

25. Starting with \( x_1 = 2 \), apply Newton’s Method once to get an approximate solution to \( x^3 - 2x - 5 = 0 \).

26. Find an equation (in any form) of the line tangent to the curve \( \mathbf{r}(t) = (t^6 + t^3)i + (t^4 + t^2)j \) at the point where \( t = 1 \).

27. A rope is attached to the bow of a boat coming in for the evening. Assume the rope is drawn in over a pulley 5 feet higher than the bow at a rate of 2 feet per second. How fast is the boat docking when the length of the rope from the bow to the pulley is 13 feet?