MATH 151 [redacted] Spring 2008

SAMPLE EXAM III

Part I - Multiple Choice

1. Given $f(x) = x^3 \ln x$, find $f'(e)$

a) e b) $3 + 3e^2$ c) e^2

d) $3e$ e) $4e^2$

2. The population of a bacteria colony triples every 5 hours. If the population follows an exponential growth model, find k.

a) $\frac{\ln 2}{5}$ b) not enough information

c) $\frac{\ln 3}{5}$ d) 3 e) $\ln 5$

3. Evaluate $\int_{-3}^{0} \sqrt{9 - x^2}\, dx$

a) $\frac{5\pi}{2}$ b) 2π c) $\frac{9\pi}{2}$

d) $\frac{9\pi}{4}$ e) None of the above

4. $\frac{d}{dx}(\tan^{-1}(x^2)) =$

a) $\frac{2x}{1 + x^4}$ b) $\frac{2}{1 + x^2}$ c) $-2x \csc(x^2) \cot(x^2)$

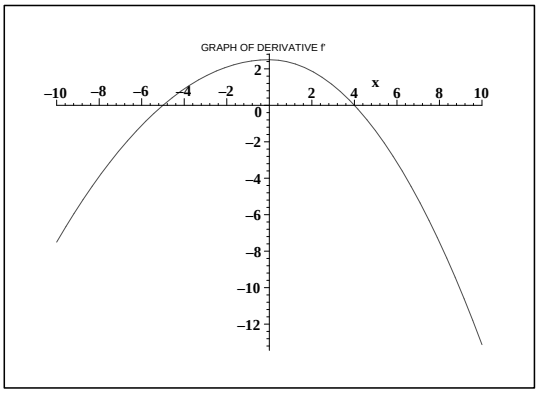
d) $2x \tan^{-1}(x^2) \sec^{-1}(x^2)$ e) $\frac{2x}{1 + x^2}$

5. Find the value of c that satisfies the Mean Value Theorem for $f(x) = x^2 + 4$ on the interval $[-1, 2]$.

a) $\frac{7}{4}$ b) $\frac{2}{3}$ c) $\frac{1}{3}$

d) 1 e) $\frac{1}{2}$

6. The graph of the *DERIVATIVE* of a function is shown below. On which intervals is the original function f concave down?
CIRCLE ALL CORRECT CHOICES-THERE MAY BE MORE THAN ONE!



a) $(-\infty, -5)$ b) $(-5, 0)$

c) $(0, 4)$ d) $(4, \infty)$

e) none of these intervals

7. Circle ALL the critical values of $f(x) = x(x-1)^{\frac{1}{3}}$
NOTE: YOU MAY CIRCLE MORE THAN ONE CHOICE!

a) 0 b) $-\frac{1}{3}$ c) $\frac{1}{4}$

d) $\frac{3}{4}$ e) 1

8. Find the absolute maximum of $f(x) = \sin x + \cos x$ on the interval $\left[0, \frac{\pi}{3}\right]$.
(**NOTE**: $\sqrt{2} \approx 1.414$ and $\sqrt{3} \approx 1.73$)

a) 1 b) 2 c) $\sqrt{2}$

d) $\frac{\sqrt{3}+1}{2}$ e) $\frac{\pi}{4}$

9. The inflection points of $f(x) = x^5 + 10x^4$ occur at which of the following?

a) $x = 6$ only b) $x = -6$ only

c) $x = 0$, $x = -8$ d) $x = 0$, $x = -6$

e) $x = 0$, $x = 6$

10. To find the rectangle of perimeter 100 cm with the largest area, you would maximize which function?

a) $f(x) = x(50 - x)$

b) $f(x) = 2x + 2(100 - x)$

c) $f(x) = \dfrac{100}{x^2}$

d) $f(x) = 50 - 2x$

e) $f(x) = 2x + \dfrac{200}{x}$

11. Which is an antiderivative of $f(x) = 2\sqrt{x} + \dfrac{1}{x^2}$?

a) $\dfrac{4}{3}x^{\frac{3}{2}} - \dfrac{1}{x} + C$

b) $\dfrac{4}{3}x^{\frac{3}{2}} - \ln(x^2) + C$

c) $\dfrac{1}{\sqrt{x}} - \dfrac{2}{x^3} + C$

d) $\dfrac{1}{\sqrt{x}} + \ln(x^2) + C$

e) $\dfrac{1}{\sqrt{x}} - \dfrac{1}{x} + C$

12. Write $\dfrac{1}{2} - \dfrac{1}{4} + \dfrac{1}{8} - \dfrac{1}{16} + \dfrac{1}{32} - \dfrac{1}{64}$ in summation notation.

a.) $\sum_{n=1}^{6} \dfrac{(-1)^{n+1}}{2^n}$

b.) $\sum_{n=1}^{6} \dfrac{(-1)^{n}}{2^n}$

c.) $\sum_{n=1}^{6} \dfrac{(-1)^{n+1}}{2n}$

d.) $\sum_{n=1}^{6} \dfrac{(-1)^{n}}{2n}$

e.) $\sum_{n=1}^{6} \dfrac{-1}{2^n}$

13. Find $\int_0^5 |x - 2|\, dx$

a.) $\dfrac{9}{2}$

b.) $\dfrac{7}{2}$

c.) $\dfrac{15}{2}$

d.) $\dfrac{11}{2}$

e.) $\dfrac{13}{2}$

14. Use the midpoint rule with $n = 4$ to approximate $\int_1^3 \ln x \, dx$

a.) $\ln \frac{3465}{256}$

b.) $\frac{1}{2} \ln \frac{3465}{256}$

c.) $\frac{1}{2} \ln \frac{32}{16}$

d.) $\ln \frac{32}{16}$

e.) None of the above.

15. Find $\lim_{x \to 0} \frac{\arctan x - x}{x^3}$

a.) $-\frac{1}{3}$

b.) $-\frac{1}{2}$

c.) 0

d.) ∞

e.) -6

16. Find an upper and lower bound on $\int_0^2 \sqrt[3]{x^2 + 1} \, dx$.

a.) $2 \leq \int_0^2 \sqrt[3]{x^2 + 1} \, dx \leq 2\sqrt[3]{5}$

b.) $1 \leq \int_0^2 \sqrt[3]{x^2 + 1} \, dx \leq \frac{1}{3}\sqrt[3]{5}$

c.) $2 \leq \int_0^2 \sqrt[3]{x^2 + 1} \, dx \leq \sqrt{5}$

d.) $1 \leq \int_0^2 \sqrt[3]{x^2 + 1} \, dx \leq \frac{1}{2}\sqrt{5}$

e.) Cannot be determined.

Part II - Work Out Problems

17. Find the derivative of $f(x) = x^{\sec x} + 2^{\arcsin x}$

18. Find $\lim_{x \to 0} (1 - x)^{\frac{5}{x}}$

19. On what intervals is $f(x) = x + 2\cos x$, $0 \leq x \leq 2\pi$, increasing?

20. Find the intervals of concavity for $f(x) = xe^{4x}$

21. Find the dimensions of the largest rectangle that can be inscribed in a circle of radius 2.

22. The acceleration of a particle is given by $\mathbf{a}(t) = (1 + e^t)\mathbf{i} + (\cos t)\mathbf{j}$. If the initial velocity is $\mathbf{i}$ and the initial position is $\mathbf{j}$, find the position of the particle at any time t.

23. Evaluate $\lim_{x \to 0} \left(\frac{1}{x^4} - \frac{1}{x^2} \right)$

24. By introducing a deadly chemical, the population of a particular bacteria culture is decreasing over time. Suppose it has been determined that the rate of change of the population at time t minutes is $\frac{1}{3}$ of the population. If the initial size of the population is 1000 bacteria, how many bacteria are present after 3 minutes?

25. Find the following:

a.) $\arccos(\frac{1}{2}) =$ ____

b.) $\sin(\arccos(-\frac{4}{5})) =$ ____

c.) $\arcsin(\sin(\frac{5\pi}{6})) =$ ____

d.) The domain of $\arccos(4x - 5) =$ _____

e.) The domain of $\arctan(\ln x) =$ _____