# Spring 2012 Math 151 

Week in Review \# 8

sections: 4.5, 4.6, 4.8
courtesy: Joe Kahlig

## Section 4.5

1. A curve passes through the point $(2,10)$ and has the property that the slope of the curve at every point $P$ is three times the y-coordinate of P. Find the equation of this curve.
2. A bacteria culture starts with 800 bacteria and will have 1000 bacteria after 30 minutes. Assume that the culture grows at a rate proportional to the number of bacteria present.
(a) Find a formula for the number of bacteria after $t$ minutes.
(b) Find the number of bacterial after 1 day.
(c) When will the population reach 3200 bacteria.
3. A chemical has a half-life of 18 days. A sample is obtained and 5 days later there remains 50 grams of the chemical.
(a) Find a formula that will give the amount of the chemical that remains $t$ days after the sample is obtained.
(b) What was the initial amount of the sample of this chemical?
(c) How long will it take until $70 \%$ of sample is gone?
4. A turkey is taken from a $350^{\circ} \mathrm{F}$ oven into a room with a temperature of $80^{\circ} \mathrm{F}$. Fifteen minutes later, the turkey is $250^{\circ}$. Use Newton's Law of cooling to solve this problem.
(a) Find a formula that will give the temperature of the turkey at time $t$.
(b) What will the temperature be after 40 minutes?

## Section 4.6

$y=\arcsin (x)=\sin ^{-1}(x)$ means $\sin y=x$ and $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$
$y=\arccos (x)=\cos ^{-1}(x)$ means $\cos y=x$ and $0 \leq y \leq \pi$
$y=\arctan (x)=\tan ^{-1}(x)$ means $\tan y=x$ and $\frac{-\pi}{2}<y<\frac{\pi}{2}$
5. Find the exact value of the following without the aid of a calculator.
(a) $\sin ^{-1}\left(\frac{-1}{2}\right)=$
(b) $\arccos \left(\frac{-\sqrt{2}}{2}\right)=$
(c) $\arctan (-\sqrt{3})=$
(d) $\tan (\arctan (1.25))=$
(e) $\sin \left(\arcsin \left(\frac{\pi}{2}\right)\right)=$
(f) $\sin ^{-1}\left(\sin \left(\frac{\pi}{2}\right)\right)=$
(g) $\sin ^{-1}\left(\sin \left(\frac{11 \pi}{9}\right)\right)=$
(h) $\arccos \left(\cos \left(\frac{16 \pi}{9}\right)\right)=$
(i) $\arctan \left(\tan \left(\frac{17 \pi}{10}\right)\right)=$
(j) $\tan \left(\arccos \left(\frac{-5}{13}\right)\right)=$
(k) $\sec \left(\arctan \left(\frac{2}{3}\right)\right)=$
(1) $\sin \left(2 \arctan \left(\frac{2}{3}\right)\right)=$
6. Write $\tan \left(\cos ^{-1} 3 x\right)$ without any trig functions.

## Derivatives of Inverse Trigonometric Functions

$$
\begin{aligned}
\frac{d}{d x} \sin ^{-1}(x) & =\frac{1}{\sqrt{1-x^{2}}} & \frac{d}{d x} \tan ^{-1}(x) & =\frac{1}{1+x^{2}} \\
\frac{d}{d x} \cos ^{-1}(x) & =\frac{-1}{\sqrt{1-x^{2}}} & \frac{d}{d x} \cot ^{-1}(x) & =\frac{-1}{1+x^{2}}
\end{aligned}
$$

7. Prove the derivative rule for $y=\tan ^{-1}(x)$.
8. Find the derivatives of the following.
(a) $y=\tan ^{-1}(5 x)$
(b) $y=x^{2} \arcsin \left(x^{2}\right)$
(c) $y=\left(\cos ^{-1}(7 x)\right)^{3}$

## Section 4.8

9. $\lim _{x \rightarrow 0} \frac{\sin (x)-x}{x^{3}}$
10. $\lim _{x \rightarrow \infty} \frac{\ln \left(x+e^{3 x}\right)}{2 x}$
11. $\lim _{x \rightarrow 0} \frac{e^{x}+e^{-x}}{x^{2}}$
12. $\lim _{x \rightarrow \infty}\left(\frac{x^{2}}{x-1}-\frac{x^{2}}{x+5}\right)$
13. $\lim _{x \rightarrow \frac{\pi}{2}}(2 x-\pi) \tan (x)$
14. $\lim _{x \rightarrow 0^{+}} x^{x}$
15. $\lim _{x \rightarrow 0}(1-5 x)^{\frac{1}{x}}$
