Spring 2012 Math 151

Week in Review # 8

sections: 4.5, 4.6, 4.8 courtesy: Joe Kahlig

Section 4.5

1. A curve passes through the point (2, 10) and has the property that the slope of the curve at every point P is three times the y-coordinate of P. Find the equation of this curve.

- 2. A bacteria culture starts with 800 bacteria and will have 1000 bacteria after 30 minutes. Assume that the culture grows at a rate proportional to the number of bacteria present.
 - (a) Find a formula for the number of bacteria after t minutes.
 - (b) Find the number of bacterial after 1 day.
 - (c) When will the population reach 3200 bacteria.

- 3. A chemical has a half-life of 18 days. A sample is obtained and 5 days later there remains 50 grams of the chemical.
 - (a) Find a formula that will give the amount of the chemical that remains t days after the sample is obtained.
 - (b) What was the initial amount of the sample of this chemical?
 - (c) How long will it take until 70% of sample is gone?

- 4. A turkey is taken from a $350^{\circ}F$ oven into a room with a temperature of $80^{\circ}F$. Fifteen minutes later, the turkey is 250° . Use Newton's Law of cooling to solve this problem.
 - (a) Find a formula that will give the temperature of the turkey at time t.
 - (b) What will the temperature be after 40 minutes?

Section 4.6

$$y = \arcsin(x) = \sin^{-1}(x) \text{ means } \sin y = x \text{ and } \frac{-\pi}{2} \le y \le \frac{\pi}{2}$$
$$y = \arccos(x) = \cos^{-1}(x) \text{ means } \cos y = x \text{ and } 0 \le y \le \pi$$
$$y = \arctan(x) = \tan^{-1}(x) \text{ means } \tan y = x \text{ and } \frac{-\pi}{2} < y < \frac{\pi}{2}$$

- 5. Find the exact value of the following without the aid of a calculator.
 - (a) $\sin^{-1}\left(\frac{-1}{2}\right) =$
 - (b) $\arccos\left(\frac{-\sqrt{2}}{2}\right) =$
 - (c) $\arctan(-\sqrt{3}) =$
 - (d) $\tan(\arctan(1.25)) =$
 - (e) $\sin\left(\arcsin\left(\frac{\pi}{2}\right)\right) =$
 - (f) $\sin^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right) =$

(g)
$$\sin^{-1}\left(\sin\left(\frac{11\pi}{9}\right)\right) =$$

(h)
$$\arccos\left(\cos\left(\frac{16\pi}{9}\right)\right) =$$

(i)
$$\arctan\left(\tan\left(\frac{17\pi}{10}\right)\right) =$$

(j)
$$\tan\left(\arccos\left(\frac{-5}{13}\right)\right) =$$

(k) $\sec\left(\arctan\left(\frac{2}{3}\right)\right) =$

(l)
$$\sin\left(2\arctan\left(\frac{2}{3}\right)\right) =$$

6. Write $\tan(\cos^{-1} 3x)$ without any trig functions.

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \qquad \frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \qquad \frac{d}{dx}\csc^{-1}(x) = \frac{-1}{x\sqrt{x^2-1}}$$
$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}} \qquad \qquad \frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1+x^2} \qquad \qquad \frac{d}{dx}\sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$$

7. Prove the derivative rule for $y = \tan^{-1}(x)$.

8. Find the derivatives of the following.

(a)
$$y = \tan^{-1}(5x)$$

(b)
$$y = x^2 \arcsin(x^2)$$

(c)
$$y = (\cos^{-1}(7x))^3$$

Section 4.8

9. $\lim_{x \to 0} \frac{\sin(x) - x}{x^3}$

10.
$$\lim_{x \to \infty} \frac{\ln(x + e^{3x})}{2x}$$

11.
$$\lim_{x \to 0} \frac{e^x + e^{-x}}{x^2}$$

12.
$$\lim_{x \to \infty} \left(\frac{x^2}{x-1} - \frac{x^2}{x+5} \right)$$

13. $\lim_{x \to \frac{\pi}{2}} (2x - \pi) \tan(x)$

14. $\lim_{x \to 0^+} x^x$

15. $\lim_{x \to 0} (1 - 5x)^{\frac{1}{x}}$