

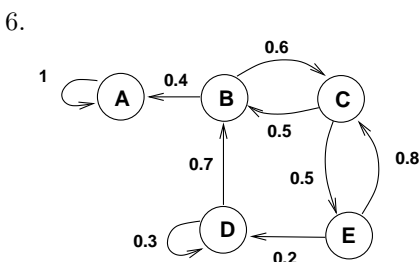
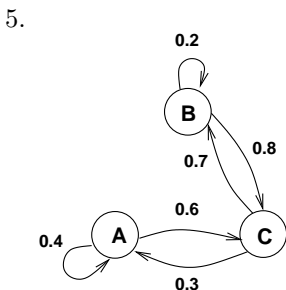
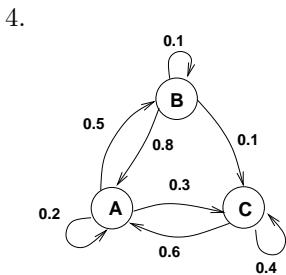
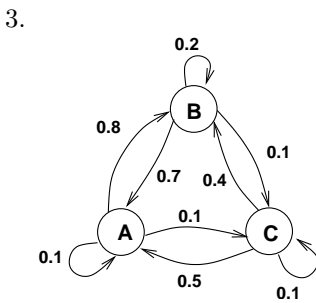
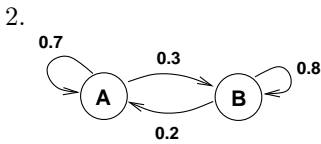
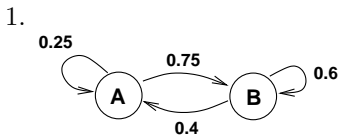
Chapter M Homework Problems

Compiled by Joe Kahlig

Section M.1

Note: Unless otherwise indicated, all transition matrices have columns labeled A, B, C,... going from left to right and rows labeled A, B, C,... going from top to bottom.

In problems 1–6, write down the transition matrix for this transition diagram.



In problems 7–12 determine if the given matrix is a transition matrix, i.e. a stochastic matrix, of a Markov process. If it is, give the associated transition diagram.

7. $T = \begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}$

8. $T = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$

9. $T = \begin{bmatrix} 0.2 & 0.3 \\ 0.3 & 0.6 \\ 0.5 & 0.1 \end{bmatrix}$

10. $T = \begin{bmatrix} 0.5 & 0.1 & 0.5 \\ 0.5 & 0.1 & 0.2 \\ 0 & 0.8 & 0.3 \end{bmatrix}$

11. $T = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.1 & 0.1 \end{bmatrix}$

12. $T = \begin{bmatrix} 0.4 & 0.2 & 0.4 \\ 0.5 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.3 \end{bmatrix}$

13. The transition matrix for a Markov process is given by

$$T = \begin{matrix} & \begin{matrix} \text{State} \\ \text{A} & \text{B} \end{matrix} \\ \begin{matrix} \text{State A} \\ \text{State B} \end{matrix} & \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} \end{matrix}$$

- (a) What does the entry $T_{1,2}$ represent?
- (b) Given that State A has occurred, what is the probability that the next outcome will be State B?
- (c) If the initial state for the Markov process is 30% in state A and 70% in state B, find the distribution vector, X_1 , and interpret the results.
- (d) What does the entry in row 2 column 2 of the matrix T^3 represent?
- (e) After three iterations of the Markov process, what percent of those who start in State A will be in State B?

14. The transition matrix for a Markov process is given by

$$T = \begin{matrix} & \begin{matrix} \text{State} \\ \text{A} & \text{B} & \text{C} \end{matrix} \\ \begin{matrix} \text{State A} \\ \text{State B} \\ \text{State C} \end{matrix} & \begin{bmatrix} 0.5 & 0.35 & 0.3 \\ 0.2 & 0.5 & 0.4 \\ 0.3 & 0.15 & 0.3 \end{bmatrix} \end{matrix}$$

- (a) What does the entry $T_{2,3}$ represent?
- (b) Given that State B has occurred, what is the probability that the next outcome will be State C?
- (c) Given that State A has occurred, what is the probability that the next outcome will be State B?

- (d) If the initial state for the Markov process is 20% in state A, 10% in state C, and 70% in state B, find the distribution vector, X_1 , and interpret the results.
- (e) What does the entry in row 2 column 3 of the matrix T^4 represent?
- (f) After four iterations of the Markov process, what percent of those who start in State B will be in State A?

In problems 15–20, use the distribution vector and the transition matrix, find the indicated distribution vector.

$$15. X_0 = \begin{bmatrix} 0.34 \\ 0.26 \\ 0.4 \end{bmatrix}, T = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.1 & 0.1 \end{bmatrix}, \text{ Find } X_3$$

$$16. X_0 = \begin{bmatrix} 0.3 \\ 0.25 \\ 0.45 \end{bmatrix}, T = \begin{bmatrix} 0.1 & 0.7 & 0.2 \\ 0.8 & 0 & 0.7 \\ 0.1 & 0.3 & 0.1 \end{bmatrix}, \text{ Find } X_4$$

$$17. X_1 = \begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix}, T = \begin{bmatrix} 0.1 & 0.3 & 0.7 \\ 0.4 & 0.2 & 0.2 \\ 0.5 & 0.5 & 0.1 \end{bmatrix}, \text{ Find } X_2$$

$$18. X_2 = \begin{bmatrix} 0.6 \\ 0.1 \\ 0.3 \end{bmatrix}, T = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.3 & 0.3 & 0.7 \\ 0.5 & 0.4 & 0.1 \end{bmatrix}, \text{ Find } X_4$$

$$19. X_3 = \begin{bmatrix} 0.6 \\ 0.1 \\ 0.3 \end{bmatrix}, T = \begin{bmatrix} 0.25 & 0.3 & 0.5 \\ 0.35 & 0.4 & 0.15 \\ 0.4 & 0.3 & 0.35 \end{bmatrix}, \text{ Find } X_6$$

$$20. X_0 = \begin{bmatrix} 0.6 \\ 0.1 \\ 0.3 \end{bmatrix}, T^2 = \begin{bmatrix} 0.25 & 0.3 & 0.5 \\ 0.35 & 0.4 & 0.15 \\ 0.4 & 0.3 & 0.35 \end{bmatrix}, \text{ Find } X_4$$

21. At a college it was noted that 12% of all business majors switched to another major the next semester, and the remaining 88% continued as business majors. Of all of the non-business majors, 25% switched to a business major the next semester and the rest did not. Assume that there are no change in majors during the summer time.

- (a) Give the transition matrix.
- (b) An entering freshman class has 40% of them being business majors and the rest are not. After 3 years, what percent of this class will be a business major?
- (c) After 3 years, what percent of those students who started as a business major will still be a business major?

22. At the beginning of 2000, the population of a certain state was 58% rural and 42% urban. Based on trends, it is expected each year that 4% of the population currently in rural areas will move into the urban areas and 2% of the population in the urban areas will move into rural areas.

- (a) Give the transition matrix.

- (b) After 3 years, what percent of those people living in rural areas moved to urban areas?
- (c) What is the distribution of the population in 2003?
- (d) What is the distribution of the population in 2015?

23. Three bookstores—the University Bookstore, Textbooks for Less, and A-plus Books—currently supply the books for a university. From a survey conducted at the beginning of the year, the University Bookstore had 40% of the market and each of the other bookstores had 30% of the market. Each semester the University Bookstore retains 80% of its customers but loses 10% to Textbooks for Less and the rest to A-plus Books. Textbooks for Less retains 70% of its customers but loses 5% to the University Bookstore and the rest to A-plus Books. A-Plus Books retains 75% of its customers but loses 20% to Textbooks for Less and the rest to the University Bookstore.

Assume that the entire summer business counts as a single semester.

- (a) Give the transition matrix.
- (b) What percent of the market share will Textbooks for Less have at the end of the year?
- (c) What will be the distribution of the market after 2 years?

24. A marketing researcher studied brand loyalty of three different brands: A, B, and C for a product. The researcher found that 97% of those who bought brand A still bought that brand one year later, while 2% had switched to brand B and 1% had switched to brand C. Of those who bought brand B, 95% still bought that brand one year later, while 2% bought brand A and the rest switched to brand C. Of those who bought brand C, 93% still bought that brand one year later, while 4% switched to brand A and 3% switched to brand B. At the start of the year, brand A had a 38% market share, brand B had 45% market share and brand C had a 17% market share

- (a) Give the transition matrix.
- (b) After 3 years, what percent of those people who started with brand B switched to brand A?
- (c) What is the distribution of the market share after 4 years?

Section M.2

In problems 25–30 determine which of these matrices are regular.

$$25. \begin{bmatrix} 0.25 & 0.4 \\ 0.75 & 0.6 \end{bmatrix}$$

$$26. \begin{bmatrix} 1 & 0.2 \\ 0 & 0.8 \end{bmatrix}$$

$$27. \begin{bmatrix} 0.2 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.4 \\ 0 & 0 & 0.1 \end{bmatrix}$$

28.
$$\begin{bmatrix} 0.2 & 0.7 & 0.5 \\ 0.8 & 0.2 & 0.4 \\ 0 & 0.1 & 0.1 \end{bmatrix}$$

29.
$$\begin{bmatrix} 0 & 0.7 & 0 \\ 0.8 & 0 & 0 \\ 0.2 & 0.3 & 1 \end{bmatrix}$$

30.
$$\begin{bmatrix} 0 & 0.7 & 0 \\ 0.8 & 0 & 1 \\ 0.2 & 0.3 & 0 \end{bmatrix}$$

In problems 31–34 find the steady-state distribution or steady-state vector for the transition matrix.

31.
$$\begin{bmatrix} 0.25 & 0.4 \\ 0.75 & 0.6 \end{bmatrix}$$

32.
$$\begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$$

33.
$$\begin{bmatrix} 0.2 & 0.7 & 0.5 \\ 0.8 & 0.2 & 0.4 \\ 0 & 0.1 & 0.1 \end{bmatrix}$$

34.
$$\begin{bmatrix} 0.5 & 0.35 & 0.3 \\ 0.2 & 0.5 & 0.4 \\ 0.3 & 0.15 & 0.3 \end{bmatrix}$$

35. For problem 22, what percent of the population will live in the rural areas when the Markov process reaches a steady-state?

36. For problem 23, find the percentage of customers that each bookstore will have when a steady-state is reached.

37. For problem 24, what is the steady-state market share for the three brands?

Section M.3

In problems 38–42, which states of the transition matrix are absorbing?

38.
$$\begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0.2 & 0 & 0.3 \\ 0.7 & 0 & 0.5 \\ 0.1 & 1 & 0.2 \end{bmatrix} \end{matrix}$$

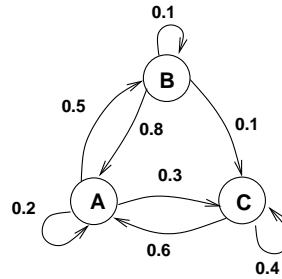
39.
$$\begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0.1 & 0 & 0.4 \\ 0.8 & 1 & 0.5 \\ 0.1 & 0 & 0.1 \end{bmatrix} \end{matrix}$$

40.
$$\begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 1 & 0.2 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0.5 & 0.3 & 0.4 \\ 0 & 0 & 0.7 & 0.6 \end{bmatrix} \end{matrix}$$

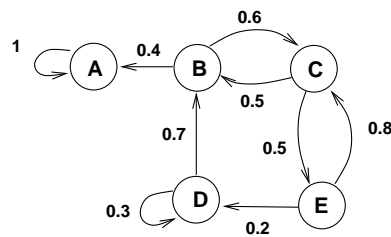
41.
$$\begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0.1 & 0 & 0 & 0.2 \\ 0.2 & 1 & 0 & 0.3 \\ 0.4 & 0 & 0 & 0.3 \\ 0.3 & 0 & 1 & 0.2 \end{bmatrix} \end{matrix}$$

42.
$$\begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0.6 & 0 & 0.7 \end{bmatrix} \end{matrix}$$

43. Which states of the transition diagram are absorbing?

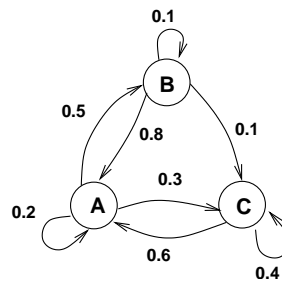


44. Which states of the transition diagram are absorbing?

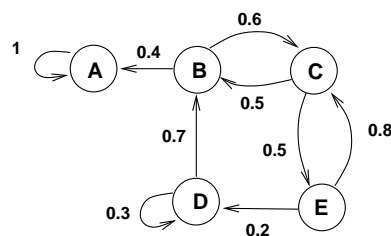


Does the transition diagram, in problems 45–49, represent an absorbing Markov process. If yes, then give the transition matrix in standard form.

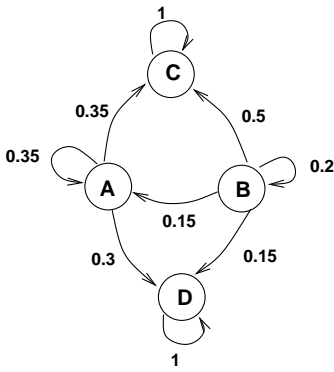
45.



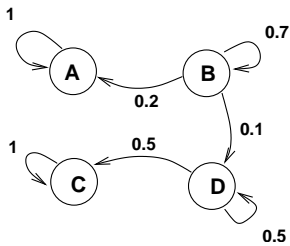
46.



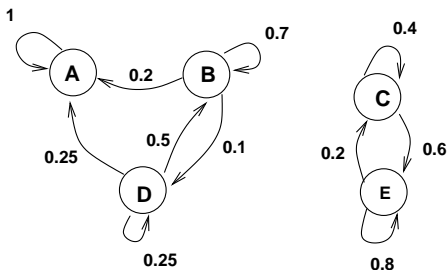
47.



48.



49.



Problems 54-58 represent a transition matrix for an absorbing Markov process. Find the limiting matrix and the fundamental matrix.

$$54. T = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 1 & 0 & 0.2 & 0.3 \\ 0 & 1 & 0.4 & 0.3 \\ 0 & 0 & 0.1 & 0.2 \\ 0 & 0 & 0.3 & 0.2 \end{bmatrix} \end{matrix}$$

$$55. T = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0.2 \\ 0 & 1 & 0 & 0.1 \\ 0 & 0 & 1 & 0.4 \\ 0 & 0 & 0 & 0.3 \end{bmatrix} \end{matrix}$$

$$56. T = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 1 & 0 & 0.3 \\ 0 & 1 & 0.2 \\ 0 & 0 & 0.5 \end{bmatrix} \end{matrix}$$

$$57. T = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 1 & 0 & 0.1 & 0.4 & 0 \\ 0 & 1 & 0.2 & 0 & 0.2 \\ 0 & 0 & 0.3 & 0.2 & 0.3 \\ 0 & 0 & 0.2 & 0.1 & 0.3 \\ 0 & 0 & 0.2 & 0.3 & 0.2 \end{bmatrix} \end{matrix}$$

$$58. T = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0.1 & 0.2 \\ 0 & 1 & 0 & 0.3 & 0 \\ 0 & 0 & 1 & 0 & 0.25 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.1 & 0.55 \end{bmatrix} \end{matrix}$$

In problems 50–53, does the transition matrix represents an absorbing Markov process? If yes, then give the transition matrix in standard form.

$$50. T = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0.6 & 0 & 0.7 \end{bmatrix} \end{matrix}$$

$$51. T = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0.1 & 0 & 0.4 \\ 0.8 & 1 & 0.5 \\ 0.1 & 0 & 0.1 \end{bmatrix} \end{matrix}$$

$$52. T = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0.1 & 0 & 0 & 0.2 \\ 0.2 & 1 & 0 & 0.3 \\ 0.4 & 0 & 1 & 0.3 \\ 0.3 & 0 & 0 & 0.2 \end{bmatrix} \end{matrix}$$

$$53. T = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0.6 \\ 0 & 0.3 & 1 & 0.4 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

59. In problem 55, what is the probability that if you start in state D you will end up in the absorbing state A? B? C?

60. Use problem 54 for these questions.

- (a) What is the probability that if you start in state C that you will end up in the absorbing state A? B?
- (b) What is the probability that if you start in state D that you will end up in the absorbing state A? B?
- (c) What is the expected number of iterations of the Markov process if you you start in State C? in State D?
- (d) If you start in state C what is the expected number of times that you will be in state C until entering an absorbing state?
- (e) If you start in state C what is the expected number of times that you will be in state D until entering an absorbing state?

61. Use problem 57 for these questions.

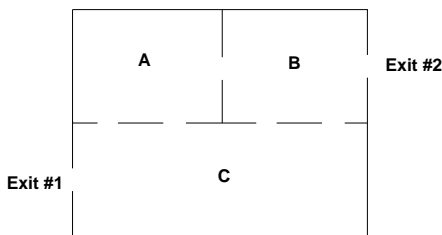
- (a) What is the probability that if you start in state C that you will end up in the absorbing state A?

- (b) What is the probability that if you start in state E that you will end up in the absorbing state B?
- (c) Which state should you start in to have the best chance of ending up in absorbing state A?
- (d) Which state should you start in to have the best chance of ending up in absorbing state B?
- (e) If you start in state C what is the expected number of times that you will be in state C until entering an absorbing state?
- (f) If you start in state D what is the expected number of times that you will be in state E until entering an absorbing state?
- (g) Which state should you start in if you want to on average end up in an absorbing state the quickest?
- (h) Which state should you start in if you want to on average end up in an absorbing state the slowest?

58% were transferred to the CCW and the remainder stayed in the ICU. Furthermore, each day 6% of the patients in CCW developed complications and were returned to the ICU, 2% died while in the CCW, 25% were released from the hospital, and the remainder stayed in the CCW.

- (a) In the long run, what percentage of the patients in the ICU are released from the hospital?
- (b) In the long run, what percentage of the patients in the CCW die without ever being released from the hospital?
- (c) What is the average number of days a patient in the ICU will stay in the hospital?

62. A mouse is placed in one of the compartments of the maze shown in the figure. The mouse studies the room and then exits through one of the doors at random. Assume that this decision process takes 2 minutes each time. The exit doors are one-way and thus once a mouse leaves the maze it can not return.



- (a) If the mouse starts in room A, what percent of the time will it exit the door in room C?
 - (b) If the mouse starts in room C, what percent of the time will it exit with Exit #2?
 - (c) Assuming that the mouse can not be started in room B, which room should the mouse be started in so that it has the best chance of using Exit #2?
 - (d) If the mouse begins in room A, what is the expected amount of time it will spend in the maze?
 - (e) If the mouse begins in room A, what is the expected amount of time it will spend in room C?
 - (f) If the mouse begins in room B, what is the expected amount of time it will spend in room A?
 - (g) What is the expected number of decisions the mouse would have to make until it exited the maze if it starts in room B?
63. After bypass surgery, patients are placed in an intensive care unit (ICU) until their conditions stabilizes. Then they are transferred to a cardiac care ward (CCW) where they remain until they are released from the hospital. In a particular area, a study of hospital records produced the following data: each day 4% of the patients in the ICU died,