Chapter M Homework Solutions
Compiled by Joe Kahlig

Note in problems 1–6 the columns are labeled in alphabetical order from the left to right.

1. $T = \begin{bmatrix} 0.25 & 0.4 \\ 0.75 & 0.6 \end{bmatrix}$

2. $T = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$

3. $T = \begin{bmatrix} 0.1 & 0.7 & 0.5 \\ 0.8 & 0.2 & 0.4 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}$

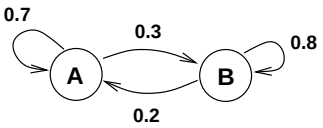
4. $T = \begin{bmatrix} 0.2 & 0.8 & 0.6 \\ 0.5 & 0.1 & 0 \\ 0.3 & 0.1 & 0.4 \end{bmatrix}$

5. $T = \begin{bmatrix} 0.4 & 0 & 0.3 \\ 0 & 0.2 & 0.7 \\ 0.6 & 0.8 & 0 \end{bmatrix}$

6. $T = \begin{bmatrix} 1 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.7 & 0 \\ 0 & 0.6 & 0 & 0 & 0.8 \\ 0 & 0 & 0 & 0.3 & 0.2 \\ 0 & 0 & 0.5 & 0 & 0 \end{bmatrix}$

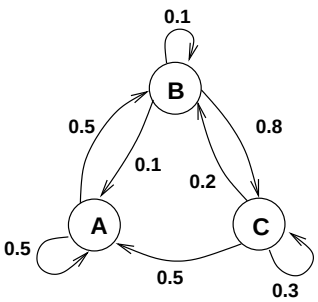
7. no, columns don't add to 1

8. yes

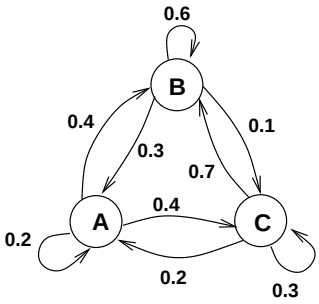


9. no, not a square matrix.

10. yes



11. yes



12. no, last column doesn't add to 1.

13. (a) $T_{1,2} = 0.2$ means that knowing that you are in state B, the probability of moving to state A is 20%

    (b) 0.6

    (c) $X_1 = TX_0 = \begin{bmatrix} 0.26 \\ 0.74 \end{bmatrix}$

    26% in state A
    74% in state B

    (d) $T^3_{2,2} = .752$ means that 75.2% of those starting in state B will still be in state B after three iterations of the Markov process.

    (e) $T^3_{2,1} = .744$
    Answer: 74.4% of those who start off in state A will be in state B.

14. (a) $T_{2,3} = 0.4$ means that knowing that you are in state C, the probability of moving to state B is 40%

    (b) 0.15

    (c) 0.2

    (d) $X_1 = TX_0 = \begin{bmatrix} 0.375 \\ 0.43 \\ 0.195 \end{bmatrix}$

    37.5% in state A
    43% in state B
    19.5% in state C

    (e) $T^4_{2,3} = .3572$ means that 35.72% of those starting in state C will be in state B after four iterations of the Markov process.

    (f) $T^4_{1,2} = 0.39755$
    Answer: 39.755% of those who start off in state B will be in state A.

15. $X_3 = T^3X_0 = \begin{bmatrix} 0.2575 \\ 0.56534 \\ 0.17716 \end{bmatrix}$

16. $X_4 = T^4X_0 = \begin{bmatrix} 0.42048 \\ 0.3783 \\ 0.20122 \end{bmatrix}$

17. $X_2 = TX_1 = \begin{bmatrix} 0.31 \\ 0.31 \\ 0.38 \end{bmatrix}$

18. $X_4 = T^2X_2 = \begin{bmatrix} 0.242 \\ 0.448 \\ 0.31 \end{bmatrix}$

19. $X_6 = T^3X_3 = \begin{bmatrix} 0.3524 \\ 0.2941 \\ 0.3534 \end{bmatrix}$

20. $X_4 = T^4X_0 = \left(T^2\right)^2 X_0 = \begin{bmatrix} 0.3585 \\ 0.2898 \\ 0.3518 \end{bmatrix}$

21. (a) State B = business major
State O = other major

$$\text{T} = \begin{array}{c} \\ \text{State B} \\ \text{State O} \end{array} \begin{array}{c} \begin{array}{cc} \text{State} & \\ \text{B} & \text{O} \end{array} \\ \begin{bmatrix} 0.88 & 0.25 \\ 0.12 & 0.75 \end{bmatrix} \end{array}$$

(b) $X_6 = T^6X_0 = \begin{bmatrix} 0.658439 \\ 0.3415605 \end{bmatrix}$

Answer: 65.8439%

(c) $T^6_{1,1} = 0.695954$

Answer: 69.5954%

22. (a) State R = rural population
State U = urban population

$$\text{T} = \begin{array}{c} \\ \text{State R} \\ \text{State U} \end{array} \begin{array}{c} \begin{array}{cc} \text{State} & \\ \text{R} & \text{U} \end{array} \\ \begin{bmatrix} 0.96 & 0.02 \\ 0.04 & 0.98 \end{bmatrix} \end{array}$$

(b) $T^3_{2,1} = 0.112944$

Answer: 11.2944%

(c) 53.8211% rural
46.1789% urban

(d) 43.0839% rural
56.9161% urban

23. (a) State U = the University Bookstore
State T = Textbooks for Less
State A = A-plus Books

$$\text{T} = \begin{array}{c} \\ \text{State U} \\ \text{State T} \\ \text{State A} \end{array} \begin{array}{c} \begin{array}{ccc} \text{State} & \text{State} & \text{State} \\ \text{U} & \text{T} & \text{A} \end{array} \\ \begin{bmatrix} 0.8 & 0.05 & 0.05 \\ 0.1 & 0.7 & 0.20 \\ 0.1 & 0.25 & 0.75 \end{bmatrix} \end{array}$$

(b) $X_3 = T^3X_0 = \begin{bmatrix} 0.284375 \\ 0.32875 \\ 0.386875 \end{bmatrix}$

Answer: 32.875

(c) $X_6 = T^6X_0 = \begin{bmatrix} 0.235596 \\ 0.346074 \\ 0.418330 \end{bmatrix}$

Answer:
23.5596% for the University Bookstore
34.6074% for Textbooks for Less
41.8330% for A-plus Books

24. (a) State A = brand A
State B = brand B
State C = brand C

$$\text{T} = \begin{array}{c} \\ \text{State A} \\ \text{State B} \\ \text{State C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{State} & \text{State} & \text{State} \\ \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 0.97 & 0.02 & 0.04 \\ 0.02 & 0.95 & 0.03 \\ 0.01 & 0.03 & 0.93 \end{bmatrix} \end{array}$$

(b) $T^3_{1,2} = 0.058752$

Answer: 5.8752%

(c) $X_4 = T^4X_0 = \begin{bmatrix} 0.396902 \\ 0.415012 \\ 0.188086 \end{bmatrix}$

Answer: 39.6902% for brand A
41.5012% for brand B
19.8086% for brand C.

25. regular

26. not regular

27. not regular

28. regular

29. not regular

30. regular

31. $X = \begin{bmatrix} \frac{8}{23} \\ \frac{15}{23} \end{bmatrix}$

32. $X = \begin{bmatrix} .4 \\ .6 \end{bmatrix}$

33. $X = \begin{bmatrix} \frac{17}{37} \\ \frac{18}{37} \\ \frac{2}{37} \end{bmatrix}$

34. $X = \begin{bmatrix} \frac{29}{73} \\ \frac{26}{73} \\ \frac{18}{73} \end{bmatrix}$

35. 33.333% will live in rural areas.

36. 20% for the University Bookstore
36% for Textbooks for Less
44% for A-plus Books

37. 48.1481% for product A
31.4815% for product B
20.3704% for product C

38. none of the states are absorbing

39. B

40. A

41. B

42. A and C

43. none

44. A

45. no, it is not an absorbing Markov process

46. yes, T =
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{bmatrix} A & B & C & D & E \\ 1 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.7 & 0 \\ 0 & 0.6 & 0 & 0 & 0.8 \\ 0 & 0 & 0 & 0.3 & 0.2 \\ 0 & 0 & 0.5 & 0 & 0 \end{bmatrix}$$

47. yes, T =
$$\begin{array}{c} \\ C \\ D \\ A \\ B \end{array}\begin{bmatrix} C & D & A & B \\ 1 & 0 & 0.35 & 0.5 \\ 0 & 1 & 0.3 & 0.15 \\ 0 & 0 & 0.35 & 0.15 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}$$

48. yes, T =
$$\begin{array}{c} \\ A \\ C \\ B \\ D \end{array}\begin{bmatrix} A & C & B & D \\ 1 & 0 & 0.2 & 0 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0.1 & 0.5 \end{bmatrix}$$

49. no, it is not an absorbing Markov process

50. no, it is not an absorbing Markov process

51. yes, T =
$$\begin{array}{c} \\ B \\ A \\ C \end{array}\begin{bmatrix} B & A & C \\ 1 & 0.8 & 0.5 \\ 0 & 0.1 & 0.4 \\ 0 & 0.1 & 0.1 \end{bmatrix}$$

52. yes, T =
$$\begin{array}{c} \\ B \\ C \\ A \\ D \end{array}\begin{bmatrix} B & C & A & D \\ 1 & 0 & 0.2 & 0.3 \\ 0 & 1 & 0.4 & 0.3 \\ 0 & 0 & 0.1 & 0.2 \\ 0 & 0 & 0.3 & 0.2 \end{bmatrix}$$

53. yes,
$$\begin{array}{c} \\ C \\ A \\ B \\ D \end{array}\begin{bmatrix} C & A & B & D \\ 1 & 0 & 0.3 & 0.4 \\ 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.6 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

54. L=
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{bmatrix} A & B & C & D \\ 1 & 0 & 0.379 & 0.470 \\ 0 & 1 & 0.621 & 0.530 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

F =
$$\begin{array}{c} \\ C \\ D \end{array}\begin{bmatrix} C & D \\ 1.212 & 0.303 \\ 0.455 & 1.264 \end{bmatrix}$$

55. L=
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{bmatrix} A & B & C & D \\ 1 & 0 & 0 & 0.286 \\ 0 & 1 & 0 & 0.143 \\ 0 & 0 & 1 & 0.571 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

F =
$$\begin{array}{c} \\ D \end{array}\begin{bmatrix} D \\ 1.429 \end{bmatrix}$$

56. L =
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{bmatrix} A & B & C \\ 1 & 0 & 0.6 \\ 0 & 1 & 0.4 \\ 0 & 0 & 0 \end{bmatrix}$$

F =
$$\begin{array}{c} \\ C \end{array}\begin{bmatrix} C \\ 2 \end{bmatrix}$$

57. L=
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{bmatrix} A & B & C & D & E \\ 1 & 0 & 0.465 & 0.692 & 0.434 \\ 0 & 1 & 0.535 & 0.308 & 0.566 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

F =
$$\begin{array}{c} \\ C \\ D \\ E \end{array}\begin{bmatrix} C & D & E \\ 1.938 & 0.769 & 1.015 \\ 0.677 & 1.538 & 0.831 \\ 0.738 & 0.769 & 1.815 \end{bmatrix}$$

58. L=
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{bmatrix} A & B & C & D & E \\ 1 & 0 & 0 & 0.289 & 0.444 \\ 0 & 1 & 0 & 0.6 & 0 \\ 0 & 0 & 1 & 0.111 & 0.556 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

F =
$$\begin{array}{c} \\ D \\ E \end{array}\begin{bmatrix} D & E \\ 2 & 0 \\ 0.444 & 2.222 \end{bmatrix}$$

59. A: 28.6%
B: 14.3%
C: 57.1%

60. (a) A: 37.9%
B: 62.1%

(b) A: 47.0%
B: 53.0%

(c) C: $1.212 + 0.455 = 1.667$ iterations
D: $0.303 + 1.264 = 1.567$ iterations

(d) 1.212 times

(e) 0.455 times

61. (a) 46.5%

(b) 56.6%

(c) D

(d) E

(e) 1.938 times

(f) 0.769 times

(g) D

(h) E

62. Find the Limiting and the fundamental matrices.

$$L= \begin{array}{c} \\ \text{E-1} \\ \text{E-2} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{bmatrix} \text{E-1} & \text{E-2} & \text{A} & \text{B} & \text{C} \\ 1 & 0 & 0.571 & 0.476 & 0.619 \\ 0 & 1 & 0.429 & 0.524 & 0.381 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \end{array} \begin{bmatrix} \text{A} & \text{B} & \text{C} \\ 3.00 & 2.00 & 2.00 \\ 2.145 & 2.619 & 1.905 \\ 2.857 & 2.381 & 3.095 \end{bmatrix}$$

(a) 57.1%

(b) 38.1%

(c) A

(d) $2 * 8 = 16$ minutes

(e) $2.857 * 2 = 5.714$ minutes

(f) 4 minutes

(g) 7

63. Find the Limiting and the fundamental matrices.

$$L= \begin{array}{c} \\ \text{die} \\ \text{home} \\ \text{ICU} \\ \text{CCW} \end{array} \begin{bmatrix} \text{die} & \text{home} & \text{ICU} & \text{CCW} \\ 1 & 0 & 0.146 & 0.087 \\ 0 & 1 & 0.854 & 0.913 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{array}{c} \\ \text{ICU} \\ \text{CCW} \end{array} \begin{bmatrix} \text{ICU} & \text{CCW} \\ 1.943 & 0.353 \\ 3.416 & 3.651 \end{bmatrix}$$

(a) 85.4%

(b) 8.7%

(c) 5.4 days