

Chapter 4 Homework Problems

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Section 4.3: Gauss Elimination for System of Linear Equations

Section 4.4: System of Linear Equations with Non-Unique Solutions

1. Perform these row operations on the matrix.

$$(a) \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 1 & -1 & 4 \\ 0 & 5 & 2 & 13 \end{array} \right] \begin{array}{l} R_1 + (-3)R_2 \rightarrow R_1 \\ R_3 + (-5)R_2 \rightarrow R_3 \end{array}$$

$$(b) \left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array} \right] \begin{array}{l} R_2 + (2)R_1 \rightarrow R_2 \\ R_3 + (-1)R_1 \rightarrow R_3 \end{array}$$

$$(c) \left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array} \right] \begin{array}{l} R_2 + (-7)R_1 \rightarrow R_2 \\ 3R_3 + (-2)R_4 \rightarrow R_3 \end{array}$$

$$(d) \left[\begin{array}{ccc|c} 1 & 3 & 5 & 2 \\ 0 & 4 & -2 & 5 \\ 3 & 2 & 16 & 10 \\ 5 & 1 & 2 & 5 \end{array} \right] \begin{array}{l} (4)R_1 + (-3)R_2 \rightarrow R_1 \\ (-3)R_4 + 5R_3 \rightarrow R_3 \end{array}$$

2. The following augmented matrices represent a system of equations. Give the solution, if any exist. Note: the variables are given in the first row of the matrix.

$$(a) \left[\begin{array}{ccc|c} x & y & z & 16 \\ 2 & 0 & 0 & 8 \\ 0 & 4 & 0 & 5 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$(b) \left[\begin{array}{ccc|c} x & y & z & 0 \\ 5 & 0 & 0 & 26 \\ 0 & 3 & 7 & 12 \\ 0 & 0 & 6 & 12 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

These word problems cover both 4.3 and 4.3.

Setup the word problems. Be sure that you define your variables as shown in class. If the solution is parametric, then tell what restrictions can be placed on the parameter.

3. A kid has 32 coins consisting of dimes and quarters. If the total value of the coins is \$5.15, how many of each does the kid have?
4. Bob has 150 coins consisting of nickels and quarters. If the total value of the coins is \$18.50, how many of each does he have?
5. You have \$50,000 to invest in Fund A and Fund B. Fund A pays 7.4% and Fund B pays 9.8%. How much do you invest in each to get a return of \$4,072 per year?

6. A bank teller has a total of 70 bills in five-, ten-, and twenty-dollar denominations. The number of fives is three times the number of tens, while the total value of the money is \$740. Find the number of each type of bill.

7. Celia had one hour to spend at the athletic club, where she will jog, play handball, and ride a bicycle. Jogging uses 11 calories per minute, handball 13 and cycling 5. She spends twice as much time jogging than riding a bicycle. How long should she participate in each of these activities in order to use 660 calories?

8. Watson Electric has production facilities in Valley Mills, marlin, and Hillsboro. Each one produces radios, stereos, and TV sets. Their production capacities are:

	Valley Mills	Marlin	Hillsboro
radios/hr	10	7	5
stereos/hr	12	10	4
TV sets/hr	6	8	13

The firm receives an order for 1365 radios, 1530 stereos, and 1890 TV sets. How many hours should each plant be scheduled in order to produce these amounts?

9. As part of a promotional campaign the T-Shirt Company packaged thousand of cartons of T-shirts for its retail outlets. Each carton contained small, medium, and large sizes. Three types of cartons were packed according to the quantities shown on the table. The entries in the table give the number (in dozens) of each size of T-shirt in the carton.

Carton	size		
	Small	Medium	Large
A	2	5	3
B	6	8	2
C	4	6	10

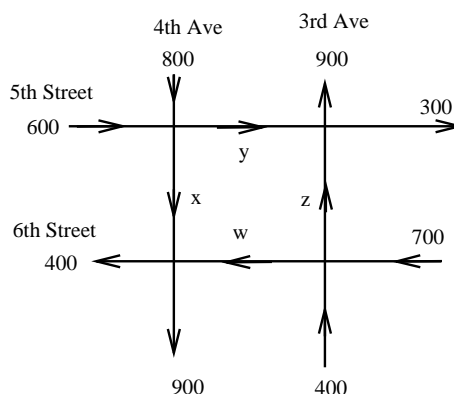
The promotion was a flop, and the company's warehouse was full of the packed cartons. When the company received an order from T-Shirt Orient for 50 dozen small, 78 dozen medium, and 52 dozen large, it wanted to fill the order with the packed cartons in order to save repacking costs. How many of each carton should it send to fill the order?

10. You own a hamburger stand and your current inventory includes 85 bread rolls, 103 beef patties, and 114 cheese slices. Your menu consists of three types of hamburgers: plain, double cheeseburger, and regular cheeseburger. Each plain hamburger require 1 beef patty and 1 bread roll. Each double cheeseburger requires 1 bread roll, 2 beef patties, and 4 cheese slices. Each regular cheese burger requires 2 cheese slices, 1 bread roll, and 1 beef patty. How many of each hamburger should you make so that all resources are used?
11. Cantwell associates, a real estate developer, is planning to build a new apartment complex consisting of one-bedroom units, and two- and three-bedroom townhouses. A total of 225 units is planned, and the number of family units(two- and three-bedroom townhouses) will be twice the number of one-bedroom units. If the number of one-bedroom units will be three times the number of three-bedroom units, find how many units of each type will be in the complex.

12. A theater has a seating capacity of 900 and charges \$2 for children, \$3 for students, and \$4 for adults. At a certain screening with full attendance, there were half as many adults as children and students combined. the receipts totaled \$2800. How many of each attended the show?
13. A farmer can buy four types of plant food. Each barrel of mix A contains 30 pounds of phosphoric acid, 50 pounds of nitrogen, and 30 pounds of potash; each barrel of mix B contains 30 pounds of phosphoric acid, 75 pounds of nitrogen, and 20 pounds of potash; each barrel of mix C contains 30 pounds of phosphoric acid, 25 pounds of nitrogen, and 20 pounds of potash; and each barrel of mix D contains 60 pounds of phosphoric acid, 25 pounds of nitrogen, and 50 pounds of potash. soil test indicate that a particular field needs 900 pounds of phosphoric acid, 750 pounds of nitrogen, and 700 pounds of potash. How many barrels of each type of food should the farmer buy to supply the necessary nutrients for the field?
14. A chemical manufacturer wants to purchase a fleet of 24 railroad tank cars with a combined carrying capacity of 250,000 gallons. Tank cars with three different carrying capacities are available: 6,000 gallons, 8,000 gallons, and 18,000 gallons. How many of each type of tank car should be purchased?
15. The Brazos Animal Shelter has been overrun with animals. To fix this problem they are having a sale: any chihuahuas (which are not dogs) costs \$4, any cat costs \$7 and any dog costs \$16. You and your friends have decided to adopt 14 of the animals. How many of each animal can be bought for 116 dollars?
16. John's T-shirt stand sells T-shirts at the following prices: A large is \$13, a medium is \$10, and a small is \$8. At the start of the day, his inventory is worth \$1940. At the end of the day he is only left with 45 shirts and these shirts are worth \$480. If during the day he sold half of the small shirts, three-fourths of the medium shirts and four-fifths of the large shirts, how many of each type of shirt does he have at the end of the day?
17. A chemical manufacturer wants to purchase a fleet of 40 railroad tank cars with a combined carrying capacity of 400,000 gallons. Tank cars with three different carrying capacities are available: 7,000 gallons, 9,000 gallons, and 20,000 gallons. How many of each type of tank car should be purchased?
18. A ball-point pen maker produces pens made of wood, silver, and gold. A wood pen requires 1 minute in a grinder and 2 minutes in a bonder, a silver pen requires .5 minutes in a grinder and 3 minutes in a bonder, and a gold pen requires 3 minutes in a grinder and 2 minutes in a bonder. If there are 200 hours of grinder time and 160 hours of bonder time available each week, how many pens of each type can be produced, assuming that all grinder and bonder time available is used?
19. Conan the Great has boasted to his hordes of followers that many a notorious villain has fallen to his awesome sword:

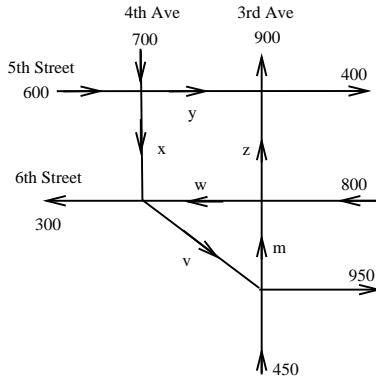
his total of 370 victims consists of evil sorcerers, warriors, and orcs. These he has slain with a total of 560 mighty thrusts of his sword; evil sorcerers require two thrusts, warriors require four thrust, and orcs each require one thrust. When asked about the number of warriors he has slain, he replies, "The number of warriors I, the mighty Conan, have slain is six times the number of evil sorcerers that have fallen to my sword!" How many of each type of villain has he slain?

20. Link has \$17,300 to invest. He decides to invest in three different companies. The QX company costs \$130 per share and pays dividends of \$1.50 per share each year. The RY company costs \$75 per share and pay dividends of \$1.00 per share each year. The KZ company costs \$90 per share and pays \$2.00 per share per year in dividends. Link wants to have twice as much money in the RY company as in the KZ company. Link also wants to earn \$251 in dividends per year. How much should Link invest in each company to meet his goals?
21. A convenience store sells 23 sodas one summer afternoon in 12-, 16-, and 20-ounce cups (small, medium, and large). The total volume of soda sold was 376 ounces. Suppose the prices for a small, medium, and large sodas are \$1, \$2, and \$3, respectively, and that the total sales were \$48. How many of each size did the store sell?
22. The figure shows the traffic flow of some roads in a certain area of a city. The arrows indicate the flow of traffic on these one-way roads and average number of vehicles entering and leaving the area is indicated on the figure. The variables x , y , z , and w are the average number of vehicles on that section of the road. Assume that the maximum capacity of each road is 1100 vehicles per hour without causing congestion.



- (a) Set up and solve the system of equations that would model this problem.
- (b) If the solution is parametric, then tell what restrictions would be placed on the parameter. If the solution is not parametric, then skip this part.
- (c) The section of **5th Street** between **4th Ave** and **3rd Ave** needs some repairs. Is there a flow pattern that will allow the smooth flow of traffic if this section of the road is reduced to an average of 150 vehicles per hour?

23. The figure shows the traffic flow for some one-way streets. The variables and numbers in the figure are the average number of cars on each section of the roads. Set up the system of equations for this problem. You do not have to solve for a solution.



- (a) Set up and solve the system of equations that would model this problem.
 (b) If the solution is parametric, then tell what restrictions would be placed on the parameter. If the solution is not parametric, then skip this part.

24. Is the matrix in row reduced (reduced row echelon) form? Note: the variables are given in the first row of the matrix.

(a)
$$\left[\begin{array}{ccc|c} x & y & z & 3 \\ 1 & 0 & 1 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

(b)
$$\left[\begin{array}{ccc|c} x & y & z & 4 \\ 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & a \end{array} \right]$$

(c)
$$\left[\begin{array}{cccc|c} x & y & z & w & a \\ 1 & 4 & 0 & -1 & a \\ 0 & 0 & 1 & 2 & b \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

25. The following augmented matrices represent a system of equations. Give the solution, if any exist. Note: the variables are given in the first row of the matrix.

(a)
$$\left[\begin{array}{cccc|c} x & y & z & w & 3 \\ 1 & 4 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

(b)
$$\left[\begin{array}{ccc|c} x & y & z & 3 \\ 1 & 0 & -1 & 3 \\ 0 & 4 & 8 & 4 \end{array} \right]$$

(c)
$$\left[\begin{array}{ccc|c} x & y & z & 0 \\ 1 & 0 & 4 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

(d)
$$\left[\begin{array}{ccc|c} x & y & z & -16 \\ 7 & 4 & -2 & -16 \\ 3 & -2 & 6 & 10 \\ 1 & 5 & 6 & 21 \\ 10 & 3 & 2 & -11 \\ 8 & -5 & 6 & -3 \end{array} \right]$$

(e)
$$\left[\begin{array}{ccc|c} x & y & z & 22 \\ 5 & 1 & -11 & 22 \\ -11 & -2 & -24 & -5 \\ 30 & 4 & -64 & 14 \\ -19 & -4 & 42 & -8 \\ 19 & 2 & -40 & 98 \end{array} \right]$$

26. Solve the system of equations by any method taught in class.

(a)
$$\begin{aligned} x + 4y - 2z &= b_1 \\ 3x - y + 7z &= b_2 \\ 2x - 5y + 9z &= b_3 \end{aligned}$$
 if

- i. $b_1 = 10, b_2 = 4, b_3 = -6$
 ii. $b_1 = -8.75, b_2 = 36.15, b_3 = 44$

(b)
$$\begin{aligned} 3x + 2y - z &= b_1 \\ 2x - 3y + z &= b_2 \\ x - y - z &= b_3 \end{aligned}$$
 if

- i. $b_1 = -49, b_2 = 131, b_3 = -7$
 ii. $b_1 = 2, b_2 = -2, b_3 = 4$

(c)
$$\begin{aligned} x + 4y - 2z &= 13 \\ 3x - y + 4z &= 6 \\ 2x - 5y + 6z &= -4 \end{aligned}$$

(d)
$$\begin{aligned} 3y + x + z &= 10 \\ 2x + 7y + 10 &= 31 + z \\ z + 4x &= 41 - 13y \end{aligned}$$

(e)
$$\begin{aligned} y + 3x &= 27 \\ x + 6z &= 35 + y \\ 3x + z &= 29 \\ 2z - y &= 39 - 4x \end{aligned}$$

(f)
$$\begin{aligned} x + y + z &= 60 \\ 5x + 8y + 11z &= 410 \\ 7x + 10y + 13z &= 530 \\ 4x + 7y + 10z &= 350 \end{aligned}$$

(g)
$$\begin{aligned} x - 2y + 5z - 2w &= 4 \\ 2x - y + z + 3w &= 31 \\ 3x + 2y + z - 5w &= -38 \\ 4x - y + 7z + w &= 35 \\ 5x - 2y + 7z - 2w &= 14 \end{aligned}$$

(h)
$$\begin{aligned} 2x + 4y - 4z - 4w &= 24 \\ 3x + 6y - 10z + 18w &= 8 \\ x + 2y - 4z + 10w &= -2 \\ 8x + 16y - 22z + 20w &= 54 \end{aligned}$$