

Chapter 4 Homework Solutions

Compiled by Joe Kahlig

$$1. \quad (a) \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & -12 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 7 & -7 \end{array} \right]$$

$$(b) \quad \left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 2 & 19 & 27 \\ 0 & 2 & -12 & -4 \end{array} \right]$$

$$(c) \quad \left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 18 & 3 & 10 \\ 3 & 0 & 6 & 1 \end{array} \right]$$

$$(d) \quad \left[\begin{array}{ccc|c} 4 & 0 & 26 & -7 \\ 0 & 4 & -2 & 5 \\ 0 & 7 & 74 & 35 \\ 5 & 1 & 2 & 5 \end{array} \right]$$

$$2. \quad (a) \quad x = 8, y = 2, \text{ and } z = 5$$

$$(b) \quad x = 0, y = 4, \text{ and } z = 2$$

3. x = the number of dimes
 y = the number of quarters

$$x + y = 32$$

$$.1x + .25y = 5.15$$

Solution:

19 dimes

13 quarters

4. X = the number of nickels
 y = the number of quarters

$$x + y = 150$$

$$.05x + .25y = 18.50$$

Solution:

95 nickels

55 quarters

5. x = amount invested in Fund A
 y = amount invested in Fund B

$$x + y = 50000$$

$$.074x + .098y = 4072$$

Solution:

\$34,500 invested in Fund A

\$15,500 invested in Fund B

6. x = number of five dollar bills
 y = number of ten dollar bills
 z = number of twenty dollar bills

$$x + y + z = 70$$

$$5x + 10y + 20z = 740$$

$$x - 3y = 0$$

Solution:

36 five dollar bills

12 ten dollar bills

22 twenty dollar bills

7. x = number of minutes jogging
 y = number of minutes playing handball
 z = number of minutes riding a bike

$$x + y + z = 60$$

$$11x + 13y + 5z = 660$$

$$x - 2z = 0$$

Solution:

20 minutes jogging

30 minutes playing handball

10 minutes riding a bike

8. x = number of hours Valley Mills is scheduled.
 y = number of hours Marlin is scheduled.
 z = number of hours Hillsboro is scheduled.

$$10x + 7y + 5z = 1365$$

$$12x + 10y + 4z = 1530$$

$$6x + 8y + 13z = 1890$$

Solution:

Valley Mills scheduled for 60 hours

Marlin scheduled for 45 hours

Hillsboro scheduled for 90 hours

9. x = the number of carton A
 y = the number of carton B
 z = the number of carton C

$$2x + 6y + 4z = 50$$

$$5x + 8y + 6z = 78$$

$$3x + 2y + 10z = 52$$

Solution:

4 of carton A

5 of carton B

3 of carton C

10. x = number of plain hamburgers
 y = number of double cheeseburgers
 z = number of regular cheeseburgers

$$x + y + z = 85$$

$$x + 2y + z = 103$$

$$4y + 2z = 114$$

Solution:

46 plain hamburgers

18 double cheeseburgers

21 regular cheeseburgers

11. x = number of one-bedroom units
 y = number of two-bedroom units
 z = number of three-bedroom units

$$x + y + z = 225$$

$$y + z = 2x$$

$$x = 3z$$

Solution: $x=75$ $y = 125$ $z = 25$

12. x = number of children at the show
 y = number of students at the show
 z = number of adults at the show

$$\begin{aligned} x + y + z &= 900 \\ 2x + 3y + 4z &= 2800 \\ 2z &= x + y \end{aligned}$$

Solution:

$$\begin{aligned} x &= 200 \\ y &= 400 \\ z &= 300 \end{aligned}$$

13. x = number of barrels of mix A
 y = number of barrels of mix B
 z = number of barrels of mix C
 w = number of barrels of mix D

$$\begin{aligned} 30x + 30y + 30z + 60w &= 900 \\ 50z + 75y + 25z + 25w &= 750 \\ 30x + 20y + 20z + 50w &= 700 \end{aligned}$$

$$\begin{array}{c} \text{Initial matrix} \\ \left[\begin{array}{cccc|c} 30 & 30 & 30 & 60 & 900 \\ 50 & 75 & 25 & 25 & 750 \\ 30 & 20 & 20 & 50 & 700 \end{array} \right] \end{array} \xrightarrow{\text{rref}} \begin{array}{c} \text{final matrix} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & -1 & -5 \\ 0 & 0 & 1 & 2 & 25 \end{array} \right] \end{array}$$

Parametric Solution:

$$\begin{aligned} x &= 10 - w \\ y &= -5 + w \\ z &= 25 - 2w \\ w &= \text{any number} \end{aligned}$$

Now place restrictions on the parameter. Since we are not told a maximum number of barrels that can be bought assume there is no limit. We know the number of barrels bought has to be non-negative.

$$\begin{aligned} x \geq 0 & & y \geq 0 & & z \geq 0 & & w \geq 0 \\ 10 - w \geq 0 & & -5 + w \geq 0 & & 25 - 2w \geq 0 & & \\ 10 \geq w & & w \geq 5 & & 25 \geq 2w & & \\ w \leq 10 & & & & 12.5 \geq w & & \\ & & & & w \leq 12.5 & & \end{aligned}$$

The restriction on the parameter is that w must be an integer and $5 \leq w \leq 10$ (i.e. $w = 5, 6, 7, 8, 9, 10$)

14. x = number of cars purchased with 6,000 gallon capacity
 y = number of cars purchased with 8,000 gallon capacity
 z = number of cars purchased with 18,000 gallon capacity

$$\begin{aligned} x + y + z &= 24 \\ 6000x + 8000y + 18000z &= 250000 \end{aligned}$$

Initial matrix and final matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 6000 & 8000 & 18000 & 250000 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -29 \\ 0 & 1 & 6 & 53 \end{array} \right]$$

Parametric solution:

$$\begin{aligned} x &= -29 + 5z \\ y &= 53 - 6z \\ z &= \text{any number} \end{aligned}$$

Once again we can not buy a part of a tank car. So z must be an integer. We also know that all of the variables must be greater than or equal to zero.

$$\begin{aligned} x \geq 0 & & y \geq 0 & & z \geq 0 \\ -29 + 5z \geq 0 & & 53 - 6z \geq 0 & & \\ 5z \geq 29 & & 53 \geq 6z & & \\ z \geq 5.8 & & 53/6 \geq z & & \\ & & z \leq \frac{53}{6} \approx 8.8333 & & \end{aligned}$$

In addition we know that the variables can not be any larger than 24

$$\begin{aligned} x \leq 24 & & y \leq 24 & & z \leq 24 \\ -29 + 5z \leq 24 & & 53 - 6z \leq 24 & & \\ 5z \leq 53 & & 29 \leq 6z & & \\ z \leq 10.6 & & 29/6 \leq z & & \\ & & z \geq \frac{29}{6} \approx 4.8333 & & \end{aligned}$$

Taken all together, we find that $z = 6, 7, 8$.

This problem ends up having only three solutions.

15. x = the number of chihuahuas bought
 y = the number of cats bought
 z = the number of dogs bought

$$\begin{aligned} x + y + z &= 14 \\ 4x + 7y + 16z &= 116 \end{aligned}$$

Solution:

$$\begin{aligned} x &= -6 + 3z \\ y &= 20 - 4z \\ z &= 2, 3, 4, \text{ or } 5 \end{aligned}$$

16. s = the number of small shirts at the end of the day.
 m = the number of medium shirts at the end of the day.
 l = the number of large shirt at the end of the day.

$$\begin{aligned} s + m + l &= 45 \\ 8s + 10m + 13l &= 480 \\ 8(2s) + 10(4m) + 13(5l) &= 1940 \end{aligned}$$

Solution:

$$\begin{aligned} &15 \text{ small shirts} \\ &10 \text{ medium shirts} \\ &20 \text{ large shirts} \end{aligned}$$

17. x = number of tank cars purchased with 7,000 gallon capacity
 y = number of tank cars purchased with 9,000 gallon capacity
 z = number of tank cars purchased with 20,000 gallon capacity

$$\begin{aligned} x + y + z &= 40 \\ 7000x + 9000y + 20000z &= 400000 \end{aligned}$$

Solution:

$$\begin{aligned} x &= -20 + 5.5z \\ y &= 60 - 6.5z \\ z &= 4, 6, \text{ or } 8 \end{aligned}$$

18. x = number of wood pens made
 y = number of silver pens made
 z = number of gold pens made

$$\begin{aligned} x + .5y + 3z &= 12000 \\ 2x + 3y + 2z &= 9600 \end{aligned}$$

Solution:

$$x = 15600 - 4z$$

$$y = -7200 + 2z$$

$3600 \leq z \leq 3900$ and z is an integer

19. x = the number of evil sorcerers slain.

y = the number of warriors slain.

z = the number of orcs slain.

$$x + y + z = 370$$

$$2x + 4y + z = 560$$

$$y = 6x$$

Solution:

10 evil sorcerers

60 warriors

300 orcs

20. x = the amount invested in the QX company

y = the amount invested in the RY company

z = the amount invested in the KZ company

$$x + y + z = 17300$$

$$2z = y$$

$$1.5 \left(\frac{x}{130} \right) + 1 \left(\frac{y}{75} \right) + 2 \left(\frac{z}{90} \right) = 251$$

Solution:

\$6,500 invested in QX

\$7,200 invested in RY

\$3,600 invested in KZ

21. x = number of 12-ounce (small) cups sold

y = number of 16-ounce (medium) cups sold

z = number of 20-ounce (large) cups sold

$$x + y + z = 23$$

$$12x + 16y + 20z = 376$$

$$x + 2y + 3z = 48$$

Solution:

$$x = z - 2$$

$$y = -2z + 25$$

$$z = 2, 3, 4, \dots, 12$$

22. (a) the variables x , y , z , and w are the average number of vehicles on that section of the road.

$$x + y = 1400$$

$$y + z = 1200$$

$$z + w = 1100$$

$$x + w = 1300$$

Solution:

$$x = 1300 - w$$

$$y = 100 + w$$

$$z = 1100 - w$$

$$w = \text{any number}$$

- (b) $200 \leq w \leq 1000$

- (c) To get this restriction we need $y = 150$. This means that $w = 50$. Since this is outside of the restrictions found in part b, the answer is no.

23. (a) $x + y = 1300$

$$y + z = 1300$$

$$z + w - m = 800$$

$$v - m = 500$$

$$x + w - v = 300$$

Solution:

columns in the matrix are x, y, z, v, m, w

$$x = 800 + m - w$$

$$y = 500 - m + w$$

$$z = 800 + m - w$$

$$v = 500 + m$$

- (b) The only restriction that we can place on the parameters m and w is that they have to be non-negative, i.e. ≥ 0 . We can not say anything else since the choice of m will affect the possible choices of w .

24. (a) yes

(b) no

(c) yes

25. (a) $x = 3 - 4y + w$

$$z = 4 - 2w$$

$y, w = \text{any number}$

- (b) $x = 3 + z$

$$y = 1 - 2z$$

$z = \text{any number}$

- (c) no solution

- (d) $x = -2, y = 1, z = 3$

- (e) no solution

26. (a) i. $x = 2 - 2z$

$$y = 2 + z$$

$z = \text{any number}$

ii. no solution

- (b) i. $x = 12, y = -22$, and $z = 41$

$$\text{ii. } x = \frac{-2}{17}, y = \frac{-10}{17}, \text{ and } z = \frac{-60}{17}$$

- (c) no solution

- (d) $x = 7 - 10z$

$$y = 1 + 3z$$

$z = \text{any number}$

- (e) $x = 8, y = 3$, and $z = 5$

- (f) $x = \frac{70}{3} + z$

$$y = \frac{110}{3} - 2z$$

$z = \text{any number}$

- (g) $x = 1, y = -2, z = 3$, and $w = 8$

- (h) $x = 26 - 2y + 14w$

$$z = 7 + 6w$$

$y = \text{any number}$

$w = \text{any number}$