

Problems:

- 1. For a continuous function f, if f'(3) = 0 and f''(3) = 7, which of these statements do we know to be true about the graph of f at x = 3?
 - (a) There is a local maximum at x = 3.
 - (b) There is an absolute maximum at x = 3.
 - (c) There is a local minimum at x = 3.
 - (d) There is an absolute minimum at x = 3.
 - (e) There is not enough information to determine the behavior of the graph at x = 3.
- 2. Find the x-values where local maximums or local minimums occur for $y = \frac{24}{x^2} + 12x + b$.
 - (a) Local min at $x = \sqrt[3]{4}$ only.
 - (b) Local max at x = 0, local min at $x = \sqrt[3]{4}$.
 - (c) Local min at x = 0, local max at $x = \sqrt[3]{4}$.
 - (d) Local max at $x = \sqrt[3]{4}$ only.
 - (e) Local max at x = 0 only.
- 3. The function f(x) is defined at all real numbers except 8 and $f'(x) = \frac{-7(x-1)(x+4)}{(x-8)^4}$. At what x-value does f(x) have a local minimum?
 - (a) x = 8 only.
 - (b) x = 1 only.
 - (c) x = -4 only.
 - (d) x = 1 and x = 8 only.
 - (e) f(x) does not have a local minimum.

4. Evaluate $\lim_{x \to 0} \frac{e^{4x} - 5 - 4x}{x^2}$

- (a) 0
- (b) 8
- (c) ∞
- (d) 2
- (e) $-\infty$
- 5. Approximate the area under the curve $f(x) = x^2 1$ on the interval [2,8] using three rectangles of equal width and midpoints.
 - (a) 106
 - (b) 226
 - (c) 80
 - (d) 304
 - (e) 160
- 6. Jeffery wants to fence a new pasture using a straight river as one side of the boundary. If Jeffery has 1200 yards of fencing materials, what are the dimensions of the largest area of the pasture that Jeffery can enclose?
 - (a) 300 yards by 300 yards
 - (b) 300 yards by 600 yards



- (c) 250 yards by 700 yards
- (d) 250 yards by 800 yards
- (e) none of above
- 7. A particle has an acceleration given by a(t) = 12t on the interval [0, 10]. If this particle has an initial velocity of 12 meters per second and has a position of 15 meters at t = 1, find the position at t = 5.
 - (a) 339 meters
 - (b) 315 meters
 - (c) 325 meters
 - (d) 311 meters
 - (e) 301 meters

8. Let f be a differentiable function such that f(3) = 1 and f'(3) = -3. If $h(x) = \frac{2f(x)}{x^2+1}$, find h'(3).

- $\begin{array}{l} \text{(a)} & -\frac{72}{100} \\ \text{(b)} & -\frac{48}{100} \\ \text{(c)} & \frac{72}{100} \\ \text{(d)} & -\frac{72}{10} \end{array}$
- (e) $\frac{48}{10}$
- 9. Find the 4003rd derivative of $g(x) = 2\sin(5x)$.
 - (a) $2 \cdot 5^{4003} \cos(5x)$
 - (b) $-2 \cdot 5^{4003} \sin(5x)$
 - (c) $2 \cdot 5^{4003} \sin(5x)$
 - (d) $2^{4003} \cdot 5^{4003} \sin(5x)$
 - (e) $-2 \cdot 5^{4003} \cos(5x)$
- 10. Sand is being dropped at a rate of 10 ft³/min onto a cone-shaped pile. If the height of the pile is always twice the base radius, at what rate is the height increasing when the pile is 8 ft high? Recall the volume formula for a cone is $V = \frac{\pi}{3}r^2h$.
 - (a) $\frac{5}{64\pi}$ ft/min
 - (b) $\frac{5}{8\pi}$ ft/min
 - (c) $\frac{5}{32\pi}$ ft/min
 - (d) $\frac{10}{9\pi}$ ft/min
 - (e) $\frac{10}{27\pi}$ ft/min
- 11. Find the value c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = -3x^2 + 5x + 5$ on the interval [0, 3].
 - (a) $\frac{17}{6}$
 - (b) 3
 - (c) $\frac{3}{2}$
 - (d) 0
 - (e) $\frac{17}{18}$

12. Use logarithmic differentiation to find the derivative of $f(x) = \frac{(x^3 + 2x)^{400}}{(1+x)^{300}}$.



$$\begin{aligned} \text{(a)} \quad f'(x) &= \left[\frac{400(3x^2+2)}{x^3+2x} - \frac{300}{1+x}\right] \cdot \frac{(x^3+2x)^{400}}{(1+x)^{300}} \\ \text{(b)} \quad f'(x) &= \left[\frac{400}{x^3+2x} - \frac{300}{1+x}\right] \cdot \frac{(x^3+2x)^{400}}{(1+x)^{300}} \\ \text{(c)} \quad f'(x) &= \frac{400(3x^2+2)}{x^3+2x} - \frac{300}{1+x} \\ \text{(d)} \quad f'(x) &= \frac{400(3x^2+2)(x^3+2x)^{398}}{(1+x)^{300}} - \frac{300(x^3+2x)^{400}}{(1+x)^{301}} \\ \text{(e)} \quad f'(x) &= \frac{100(x^3+2x)^{398}(9x^3+12x^2+2x+8)}{(1+x)^{301}} \end{aligned}$$

13. Find the t-values where the tangent line to the following parametrically defined curve is horizontal or vertical.

$$x = 2t^3 - t^2 + 6$$
 and $y = -t^3 + \frac{9}{2}t^2 - 6t$

- (a) horizontal tangents occur at t = 1, 2; vertical tangents occur at $t = 0, \frac{1}{3}$
- (b) horizontal tangents occur at $t = 0, \frac{1}{3}$; vertical tangents occur at t = 1, 2
- (c) horizontal tangents occur at $t = \frac{2}{3}$, 1; vertical tangents occur at t = 0
- (d) horizontal tangents occur at t = 0; vertical tangents occur at $t = \frac{2}{3}, 1$
- (e) horizontal tangents occur at t = 1; there is no vertical tangent

14. Which of the following is a vector of unit length tangent to $\langle \sqrt{10t+5}, e^{4t-8} \rangle$ at t=2?

- (a) $\left< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right>$
- (b) $\left< \frac{3}{5}, \frac{4}{5} \right>$
- (c) $\left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle$
- (d) $\langle 1, 1 \rangle$
- $(\alpha) (1, 1)$
- (e) $\langle 1, \frac{4}{3} \rangle$

15. Find the values of the constants a and b that make the following piecewise function differentiable everywhere:

$$f(x) = \begin{cases} ax^3 + 16x & \text{if } x < 1\\ 5x^2 + b & \text{if } x \ge 1 \end{cases}$$

- (a) a = 16, b = 5(b) a = -2, b = 9(c) $a = -\frac{5}{3}, b = \frac{28}{3}$ (d) a = 1, b = 0(e) there is not enough information to determine *a* and *b* 16. Suppose $\int_{5}^{9} g(x)dx = 4$. Evaluate $\int_{5}^{9} (3 - 4g(x))dx$. (a) 39 (b) -13
 - (c) 19
 - (d) -4

$$(a) = 1$$

17. Let $\int_{\tan x}^{x} \frac{1}{\sqrt{4+t^3}} dt$. Find f'(x).



(a)
$$f'(x) = -\frac{\tan(x)}{\sqrt{4} + \tan^3(x)} + \frac{1}{\sqrt{4} + x^3}$$

(b) $f'(x) = \frac{\sec^2(x)}{\sqrt{4} + \tan^3(x)} + \frac{1}{\sqrt{4} + x^3}$
(c) $f'(x) = -\frac{\sec^2(x)}{\sqrt{4} + \tan^3(x)} + \frac{1}{\sqrt{4} + x^3}$
(d) $f'(x) = \frac{\tan(x)}{\sqrt{4} + \tan^3(x)} + \frac{1}{\sqrt{4} + x^3}$
(e) $f'(x)$ does not exist
18. Evaluate $\int_1^2 \left(\frac{9}{x^5} - \frac{2}{x}\right) dx$.
(a) $\frac{135}{64} - 2\ln(2)$
(b) $-\frac{135}{64} - 2\ln(2)$
(c) $\frac{135}{64} + 2\ln(2)$
(d) $-\frac{135}{64} - 2\ln(2)$
(e) $\frac{135}{64} + \ln(2)$
19. Compute $\int \left(3x^2 - 10 + \frac{3}{x^2 + 1}\right) dx$.
(a) $\frac{x^3}{3} - 10x + 3\arctan(x) + C$
(b) $\frac{x^3}{3} - 10x + 3\arctan(x) + C$
(c) $x^3 - 10x + 3\arctan(x) + C$
(d) $x^3 - 10x + 3\arctan(x) + C$
(e) $\frac{x^3}{3} - 10x + 3\arctan(x) + C$
(f) $x^3 - 10x + 3\arctan(x) + C$
(g) $\frac{x^3}{3} - 10x + 3\arctan(x) + C$
(h) $x^3 - 10x + 3\tanh(x) + C$

- 20. The velocity function, in meters per second, is v(t) = 3t 7. What is the displacement of the particle in the first four seconds it moves?
 - (a) 4 m
 - (b) -32 m
 - (c) 32 m
 - (d) 12 m
 - (e) -4 m
 - 21. A plane is flying at 850 mph at N45°E. The wind is blowing at 30mph S60°E. Find the true direction of the plane.

(a)
$$\theta = \arctan\left(\frac{425\sqrt{2} + 15\sqrt{3}}{425\sqrt{2} - 15}\right)$$

(b) $\theta = \arctan\left(\frac{425\sqrt{2} + 15\sqrt{3}}{425\sqrt{2} + 15}\right)$
(c) $\theta = \arctan\left(\frac{15 - 425\sqrt{2}}{15\sqrt{3} + 425\sqrt{2}}\right)$
(d) $\theta = \arctan\left(\frac{425\sqrt{2} - 15}{425\sqrt{2} + 15\sqrt{3}}\right)$



(e)
$$\theta = \arctan\left(\frac{15\sqrt{3} + 425\sqrt{2}}{15 - 425\sqrt{2}}\right)$$

22. A horizontal force of 20 pounds is acting on a box as it is pushed up a ramp that is 5 feet long and inclined at an angle of 60° above the horizontal. Find the work done on the box.

- (a) 50 ft-lb
- (b) $50\sqrt{3}$ ft-lb
- (c) $50\sqrt{2}$ ft-lb
- (d) 100 ft-lb
- (e) 10 ft-lb

23. Given the points A(1,0), B(0,2) and C(3,4), find the angle $\theta = \angle BAC$.

(a)
$$\theta = \arccos\left(\frac{3}{5}\right)$$

(b) $\theta = \arccos\left(-\frac{1}{\sqrt{65}}\right)$
(c) $\theta = 180^{\circ}$
(d) $\theta = \arccos\left(\frac{1}{\sqrt{65}}\right)$
(e) $\theta = \arccos\left(\frac{3}{\sqrt{17}}\right)$

24. Find the slope of the tangent line to the graph $x^2y^2 - 3y = 0$ at the point (1, -3).

- (a) $-\frac{5}{2}$ (b) $\frac{5}{2}$ (c) -8(d) 2 (e) -225. Compute $\lim_{x \to 3^+} \frac{x^2 - 2x - 7}{x^2 - 5x + 6}$ (a) $-\infty$ (b) ∞ (c) 1
 - (d) -4
 - (e) $-\frac{7}{6}$

26. Use linear approximation to estimate $\sqrt[3]{27.2}$

 $\begin{array}{ll} \text{(a)} & \frac{801}{270} \\ \text{(b)} & \frac{1}{135} \\ \text{(c)} & \frac{406}{135} \\ \text{(d)} & \frac{402}{135} \\ \text{(e)} & \frac{803}{270} \end{array}$ $27. \text{ Given } f(x) = \begin{cases} x+2 & \text{if } x < 2 \\ x^2 & \text{if } x = 2 \\ 5 & \text{if } x > 2 \end{cases}. \text{ Which of the following statements is true?}$



- (a) f(x) is continuous from the left at x = 2.
- (b) f(x) is continuous from the right at x = 2.
- (c) f(x) is continuous at x = 2.
- (d) f(x) is not continuous at x = 2 because $\lim_{x \to 2} f(x)$ exists but does not equal to f(2).
- (e) None of above.

28. Compute $\lim_{x \to -\infty} \frac{5e^{2x} - 8e^{-3x}}{3e^{2x} + 2e^{-3x}}.$

- (a) 0
- (b) $\frac{5}{3}$
- (c) -4
- (d) $-\infty$
- (e) ∞
- 29. The domain of f(x) is all real numbers and $f''(x) = 3x(x^2-16)(x-4)$. Give the x-coordinate of the inflection point(s).
 - (a) x = 0, x = 4, and x = -4
 - (b) x = 0 and x = -4 only
 - (c) x = 4 and x = -4 only
 - (d) x = 0 and x = 4 only
 - (e) f(x) has no inflection points.

30. Find the derivative of the function $f(x) = \arcsin(e^{4x})$.

(a)
$$f'(x) = \frac{4e^{4x}}{\sqrt{1 - e^{8x}}}$$

(b) $f'(x) = -\frac{4e^{4x}}{\sqrt{1 - e^{8x}}}$
(c) $f'(x) = \frac{4e^{4x}}{1 + e^{8x}}$
(d) $f'(x) = -\frac{4e^{4x}}{1 + e^{8x}}$
(e) $f'(x) = \frac{4e^{4x}}{1 - e^{8x}}$