Exam 3 Review (Sections 4.1 - 5.2)

Problem 1. Find the critical numbers for the function $f(x) = (x-2)^{\frac{1}{7}}x^2$.

Problem 2. Find the inflection points for the function $f(x) = x^5 + 10x^4$.

Problem 3. Given the function $f(x) = \dfrac{\sqrt{x-4}}{x+1}$, find the following:

a) Domain

b) Vertical asymptotes

c) Horizontal asymptotes

d) $x-$intercept(s) and $y-$intercept.

e) Critical numbers and intervals where the function is increasing and decreasing.

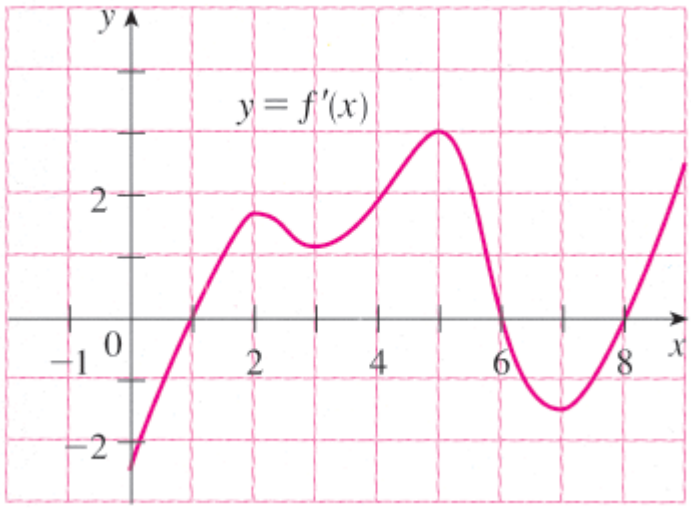
f) Inflection points and intervals where the function is concave up and concave down.

Problem 4. Find the absolute maxima and the absolute minima for the following:

a) $f(x) = (5 + \ln x)^4, \quad [\frac{1}{e^7}, 1]$.

b) $f(x) = \sin^2 x + \cos^2 x, \quad [0, \frac{3\pi}{4}]$.

Problem 5. The graph of the first derivative $f'(x)$ of a function $f(x)$ is shown.



a) On what intervals is f increasing? Decreasing?

b) Where does f have a local maxima? A local minima?

c) On what intervals is f concave up? Concave down?

d) What are the inflection points of f?

Problem 6. A box with an open top has a volume of 400 cubic meters. If the height of the box is twice its width, find the dimensions of a box which would have the smallest possible surface area.

Problem 7. Find the point on the parabola $y = \frac{1}{2}x^2$ that is closest to the point $(5, 1)$.

Problem 8. Find the following limits:

a) $\lim_{x \to 0^+} (\sin x)^{\tan x}$

b) $\lim_{x \to \infty} 5\left(1 + \frac{3}{x}\right)^{2x}$

c) $\lim_{x \to 1^+} (x-1) \tan\left(\frac{\pi x}{2}\right)$

d) $\lim_{x \to \infty} \arcsin\left(\frac{1-3x^2}{6x^2-x}\right)$

Problem 9. Given $f(3) = 0, \quad f'(3) = 0, \quad f''(3) = -4$ what can you say about $f(x)$ at $x = 3$?

Problem 10. Find the value of c that satisfies the Mean value Theorem for $f(x) = \ln(x^3), \quad [1, 4]$.

Problem 11. Find the most general antiderivative for $f(x) = 15x^4 + \sqrt[3]{x^2} + \dfrac{\pi}{x} + \dfrac{2}{x^2} + \dfrac{5}{1+x^2} + 9\sec^2 x$.

Problem 12. Find $f(x)$ if $f''(x) = 4x^3 + 2\cos x, \quad f(0) = 0, \ f'(0) = 4$

Problem 13. Use geometry to evaluate $\int_{-1}^{2} (1 - x)\, dx$.

Problem 14. Approximate the area under the curve $f(x) = e^{x^2}$ in the interval $[0, 1]$ using $n = 4$ and the midpoint rule.

Problem 15. Express the area under the curve $f(x) = \dfrac{2x}{x^2+1}$ on the interval $[1, 3]$ as a limit.

Problem 16. Evaluate the following definite integrals.

a) $\displaystyle\int_1^4 \frac{x^2-1}{x\sqrt{x}}\,dx$

b) $\displaystyle\int_1^2 \left(\frac{3}{x} + 2^x\right)\,dx$