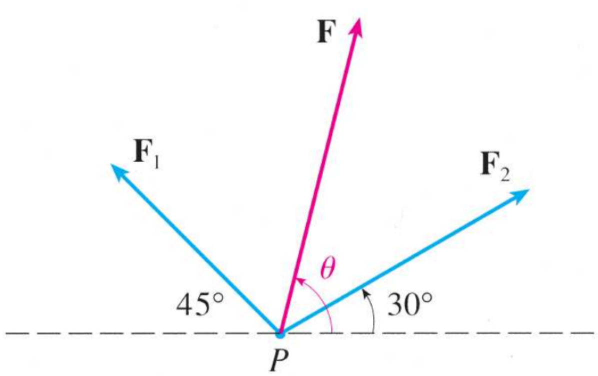
FINAL EXAM REVIEW

Problem 1. If $\vec{a} = \langle 4, 3 \rangle$ and $\vec{b} = \langle -2, 1 \rangle$, find the unit vector $\vec{a}$ and the vector $(5\vec{a} - 2\vec{b})$.

Problem 2. Two forces $\vec{F}1$ and $\vec{F}2$, with magnitudes 10 lbs and 12 lbs respectively, act on an object P as shown in the diagram below. Find the direction and magnitude of the resultant force $\vec{F}$ acting on P.



Problem 3. Find the vector projection of $\langle 4, 8 \rangle$ onto $\langle 2, 1 \rangle$.

Problem 4. A parametric curve is given by $x = 2\sin\theta$, $y = 3\cos\theta$. Find its Cartesian equation.

Problem 5. Find the vector equation of a line passing through the points $(1, 2)$ and $(-1, 4)$.

Problem 6. Evaluate the following limits:

a) $\lim_{x \to 2^-} \dfrac{x-1}{x^2(x+2)}$

b) $\lim_{x \to -3} \dfrac{x^2 - x - 12}{x+3}$

c) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2+4x}}{4x+1}$

d) $\lim_{x \to \infty} \dfrac{e^x - e^{-3x}}{e^{3x} + e^{-3x}}$

e) $\lim_{x \to \infty} [\ln(3x^2) - \ln(6x^4 - 3x + 1)]$

Problem 7. Find the derivative or $\dfrac{dy}{dx}$ for the following:

a) $y = x^3(4x^5 + 5)^8$

b) $f(x) = \sqrt{\cos(\sin^2 x)}$

c) $y = \ln(xe^{-x})$

d) $e^y \sin x = x + xy$

Problem 8. Find the equation of the tangent line for the curve given by $x = 1 - t^3,\ y = t^2 - 3t + 1$ at $t = 2$.

Problem 9. Differentiate $y = \dfrac{e^x(x^2+2)^3}{(x+1)^4(x^2+5)^2}$

Problem 10. Differentiate $y = (4x^2 - x + 1)^{\sin x}$

Problem 11. A culture starts with 1000 bacteria and the population triples every half hour.

a) Find an expression for the number of bacteria after t hours.

b) Find the number of bateria present in the sample after 20 minutes.

Problem 12. The length of a rectangle is decreasing at a rate of $1m/s$ while the area of the rectangle remains constant. How fast is the width of the rectangle increasing when its length is $10m$ and its width is $5m$?

Problem 13. Find the values of a and c which would make $f(x)$ continuous and differentiable at $x = 3$.

$$f(x) = \begin{cases} ax^2 - 9x + c & \text{if } x < 3 \\ 2ax^2 + a^2x - 5 & \text{if } x \geq 3 \end{cases}$$

Problem 14. Use linear approximation to find an approximate value for $(1.97)^6$.

Problem 15. Find the absolute maxima and the absolute minima for $f(x) = 1 + 27x - x^3$ on the interval $[0, 4]$.

Problem 16. Find the intervals on which $f(x) = x^5 - 15x^4 + 6$ is decreasing and concave down.

Problem 17. Find the following limits.

a) $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$

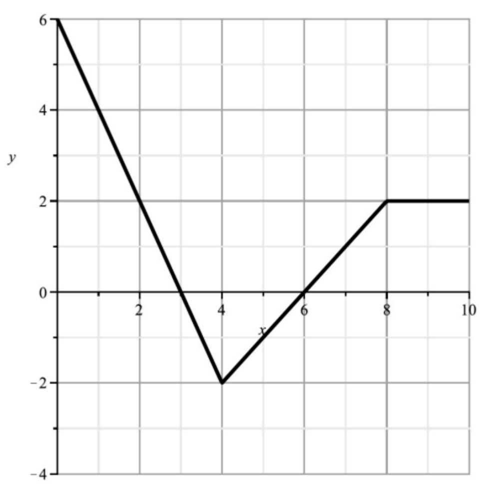
b) $\lim_{x \to 0^+} x \ln x$

c) $\lim_{x \to 1} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1} \right)$

d) $\lim_{x \to \infty} x^{3/x}$

Problem 18. Approximate the area under the curve $f(x) = x^2 + 1$ on the interval $[1, 7]$ using 3 rectangles of equal width and the left endpoints of each rectangle.

Problem 19. If $g(x) = \int_0^x f(t)\,dt$, where the graph of $f(t)$ is given below for $0 \leq x \leq 10$, evaluate $g(4)$ and $g(10)$. Where is $g(x)$ decreasing?



Problem 20. Find $g'(x)$ if $g(x) = \displaystyle\int_{x^2}^{\sin x} \frac{\cos t}{t}\, dt$.

Problem 21. Evaluate the following integrals:

a) $\displaystyle\int \frac{1+\sqrt{x}+x}{x}\, dx$

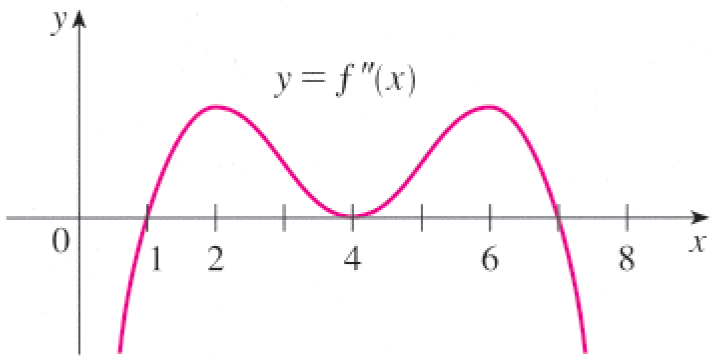
b) $\displaystyle\int \left(2x^3 - 6x + \frac{3}{x^2+1}\right)\, dx$

c) $\int_0^{\pi} \left(5e^x + \sin(2x)\right)\, dx$

d) $\int x^3\sqrt{1+x^2}\, dx$

e) $\int_1^e \frac{\ln x}{x}\, dx$

Problem 22. The graph of the second derivative $f''(x)$ of a function $f(x)$ is shown.



a) On what intervals is f concave up?

b) On what intervals is f concave down?

c) What are the inflection points of f?

d) Where does f have a local maxima? A local minima?