Math 151 WIR 1

Week-in-Review 1 (Vectors part 1)

$\langle -1, 2\rangle$ final

Problem 1. A vector starts at the initial point $A(-1, 2)$ and ends at $B(-3, 5)$. $\langle -3, 5\rangle$.

(1) Draw the vector. Then sketch the position vector.
(2) Find the vector $\vec{AB}$.
(3) Find the magnitude of $\vec{AB}$ and the unit vector of $\vec{AB}$.
(4) What is vector $\vec{BA}$?

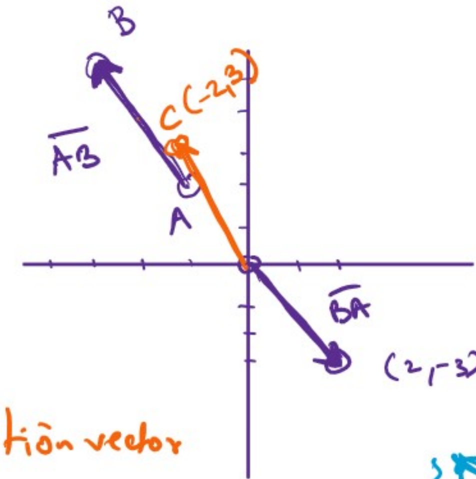
(2) $\underset{B}{\text{final}} - \underset{A}{\text{initial}}$

$= (-3,5) - (-1,2)$

$= (-3-(-1), 5-2)$

$= (-2, 3)$

$\boxed{\vec{AB} = \langle -2, 3\rangle} \rightarrow$ position vector



(3) magnitude is length (Pythagoras theorem)

$|\vec{AB}| = \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \boxed{\sqrt{13}}$

direction { unit vector $\vec{AB}$ $\rightarrow$ vector of length 1 along $\vec{AB}$

unit vector $\vec{AB} = \dfrac{\vec{AB}}{|\vec{AB}|} = \dfrac{\langle -2, 3\rangle}{\sqrt{13}}$

$= \left\langle -\dfrac{2}{\sqrt{13}}, \dfrac{3}{\sqrt{13}} \right\rangle$

(4) vector $\vec{BA} = \underset{A}{\text{final}} - \underset{B}{\text{initial}}$

$= (-1, 2) - (-3, 5)$

$$= (-1, 2) - (-3, 5)$$

$$= (-1+3,\ 2-5)$$

$$\boxed{\overrightarrow{BA} = \langle 2, -3 \rangle}$$

$$= -\langle -2, 3 \rangle$$

$$= -\overrightarrow{AB}$$

$$\overrightarrow{AB} \neq \overline{BA}$$

$$\overline{AB} = -\overline{BA}$$

Problem 2. Given two vectors $\vec{a} = 2\vec{i} + 3\vec{j}$ and $\vec{b} = 3\vec{i} - 2\vec{j}$, find the following

(1) $|\vec{a} - \vec{b}|$

(2) $3\vec{a} + 4\vec{b} - \vec{i}$

(3) A [illegible] vector of length 5 in the direction of $\vec{a}$.

(4) A unit vector in the direction opposite to $\vec{b}$.

Basis vectors.

$\hat{i} = \langle 1, 0 \rangle$

$\hat{j} = \langle 0, 1 \rangle$

$$\vec{a} = 2i + 3\hat{j} = \langle 2, 3 \rangle$$

$$\vec{b} = 3\hat{i} - 2\hat{j} = \langle 3, -2 \rangle$$

① $|\bar{a} - \bar{b}| \longleftarrow$ length of vector $(\bar{a} - \bar{b})$

a) $\bar{a} - \bar{b} = \langle 2, 3 \rangle - \langle 3, -2 \rangle$

$= \langle -1, 5 \rangle = \bar{c}$

b) $|\bar{a} - \bar{b}| = \sqrt{(-1)^2 + (5)^2} = \sqrt{26}$

② $3\bar{a} + 4\bar{b} - \hat{i}$

$= 3\langle 2, 3 \rangle + 4\langle 3, -2 \rangle - \langle 1, 0 \rangle$

$= \langle 6, 9 \rangle + \langle 12, -8 \rangle - \langle 1, 0 \rangle$

$= \langle (6 + 12 - 1), (9 - 8 - 0) \rangle$

$= \langle 17, 1 \rangle$ Ans.

③ $\vec{a} = \langle 2, 3 \rangle$

a) unit vector $\vec{a} = \dfrac{\langle 2, 3 \rangle}{\sqrt{4 + 9}} = \dfrac{\langle 2, 3 \rangle}{\sqrt{13}}$

$= \left\langle \dfrac{2}{\sqrt{13}}, \dfrac{3}{\sqrt{13}} \right\rangle$

$= \left\langle \frac{}{\sqrt{13}}, \frac{}{\sqrt{13}} \right.$

b) vector of length 5 along $\vec{a}$

$(\text{unit vector } \vec{a}) * 5$

$= 5 \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$

$= \left\langle \frac{10}{\sqrt{13}}, \frac{15}{\sqrt{13}} \right\rangle$

4) $\vec{b} = \langle 3, -2 \rangle$

unit vector $\vec{b} = \dfrac{\langle 3, -2 \rangle}{\sqrt{9+4}} = \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$

unit vector in opposite direction $=$

$(-1) \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$

$= \left\langle \frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$

unit vector $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$\rightarrow$ This is direction along $\vec{a}$

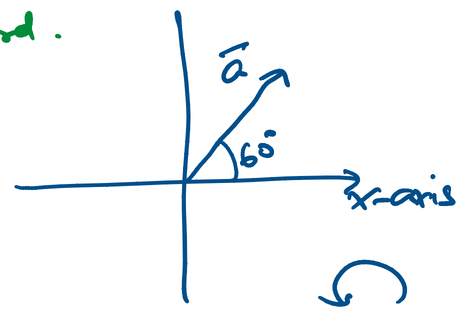
Problem 3. .

θ = angle made with (+)ve x-axis

(1) Find the ~~direction~~ angle that the vector $\vec{a} =< 1, \sqrt{3} >$ makes with the positive $x-$axis.
(2) Find the direction that the vector $\vec{b} =< -1, \sqrt{3} >$ makes with the positive $x-$axis.
(3) Find the direction that the vector $\vec{c} =< -1, -\sqrt{3} >$ makes with the positive $x-$axis.
(4) Find the direction that the vector $\vec{d} =< 1, -\sqrt{3} >$ makes with the positive $x-$axis.

(1) $\vec{a} = \langle 1, \sqrt{3} \rangle$

$$m = \tan\theta = \frac{\text{rise}}{\text{run}} = \frac{y\text{ coord}}{x\text{ coord}}.$$



$$\theta = \arctan\left(\frac{\sqrt{3}}{1}\right)$$
$$= \arctan(\sqrt{3})$$
$$= 60^\circ$$

vector $\vec{a}$ makes an angle of 60° with (+)ve x-axis

UNIT CIRCLE

fn \ θ	0	30°	45°	60°	90°
	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

$\frac{\sin\theta}{\cos\theta} = \tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	DNE ∞

If $\tan(60^\circ) = \sqrt{3}$

$\arctan(\sqrt{3}) = 60^\circ$

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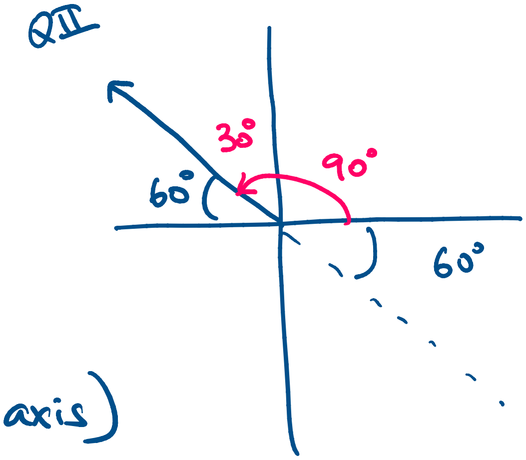
(2) $\vec{b} = \langle -1, \sqrt{3} \rangle$



$\theta = \arctan\left(\frac{\sqrt{3}}{-1}\right)$

$= \arctan(-\sqrt{3})$

$= -60^\circ$ (below x-axis)

angle that $\vec{b}$ makes with (+)ve x-axis

$= 90^\circ + 30^\circ = 120^\circ$

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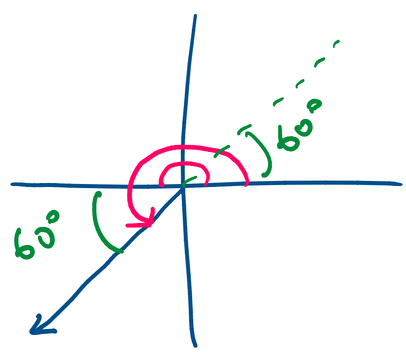
(3) $\vec{c} = \langle -1, -\sqrt{3} \rangle$



$\theta = \arctan\left(\frac{-\sqrt{3}}{-1}\right)$

$= \arctan(\sqrt{3})$

$= 60^\circ$

angle made with (+)ve x-axis $= 60^\circ + 180^\circ$

angle made with (+)ve x-axis $= 60^\circ + 180^\circ$

$= 240^\circ$

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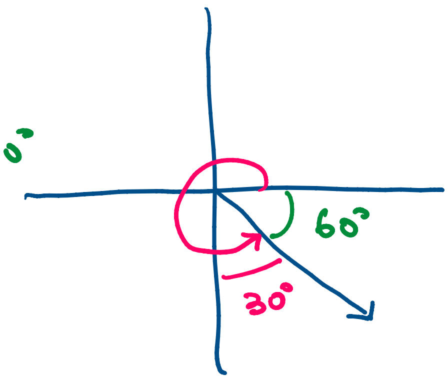
(4) $\vec{d} = \langle 1, -\sqrt{3} \rangle$

$\theta = \arctan\left(\frac{-\sqrt{3}}{1}\right) = -60^\circ$



angle $= 270^\circ + 30^\circ$

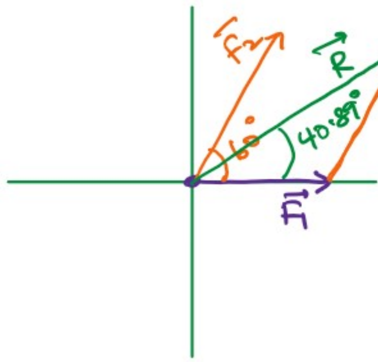
$= 300^\circ$

Problem 4. Two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$, are acting on an object P. $\mathbf{F_1}$ has a magnitude of 2 lbs and acts along the positive $x-$axis while $\mathbf{F_2}$ has a magnitude of 4 lbs and acts at an angle of $60°$ with respect to the positive $x-$axis. Find the magnitude and direction of the resultant force acting on P.

$\vec{R}$

$|\vec{F_1}| = 2$ lbs

$\vec{F_1} = \langle 2, 0 \rangle$



$|\vec{F_2}| = 4$ lbs

$$\vec{F_2} = \langle 4\cos(60°), 4\sin 60° \rangle$$
$$= \langle 4\left(\tfrac{1}{2}\right), 4\left(\tfrac{\sqrt{3}}{2}\right) \rangle$$
$$= \langle 2, 2\sqrt{3} \rangle$$

Resultant force $\vec{R} = \vec{F_1} + \vec{F_2}$

$$= \langle 2, 0 \rangle + \langle 2, 2\sqrt{3} \rangle$$

$$\vec{R} = \langle 4, 2\sqrt{3} \rangle$$

a) magnitude $= |\vec{R}| = \sqrt{(4)^2 + (2\sqrt{3})^2}$

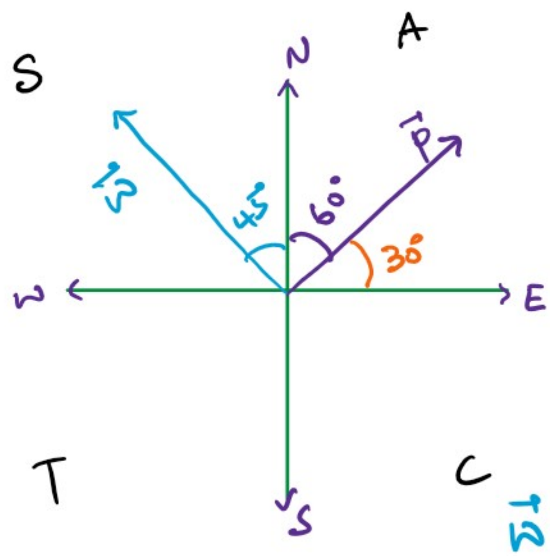
$$= \sqrt{16 + 12}$$
$$= \sqrt{28}$$

b) direction : $\theta = \arctan\left(\frac{2\sqrt{3}}{4}\right)$

$$= \arctan\left(\frac{\sqrt{3}}{2}\right)$$
$$= \arctan(0.866)$$
$$= 40.89° \text{ from (+)ve x-axis}$$

Problem 5. A pilot steers his plane in the direction **N60E** at a speed of 250 kilometers per hour while the wind is blowing in the direction **N45W** at a speed of 50 kilometers per hour. Find the true course (in bearings) and the ground speed of the plane.

$\vec{w}$ $\vec{p}$ $\vec{R}$



$$|\vec{p}| = 250$$

$$\vec{p} = \langle (250)\cos 30^\circ, (250)\sin 30^\circ \rangle$$

$$= \langle 250\left(\tfrac{\sqrt{3}}{2}\right), 250\left(\tfrac{1}{2}\right) \rangle$$

$$\boxed{\vec{p} = \langle 125\sqrt{3}, 125 \rangle}$$

$$|\vec{w}| = 50$$

$$\vec{w} = \langle 50\cos 135^\circ, 50\sin 135^\circ \rangle.$$

$$= \langle -50\left(\tfrac{\sqrt{2}}{2}\right), 50\tfrac{\sqrt{2}}{2} \rangle$$

$$= \langle -25\sqrt{2}, 25\sqrt{2} \rangle$$

$$\vec{R} = \vec{p} + \vec{w}$$

$$= \langle 125\sqrt{3}, 125 \rangle + \langle -25\sqrt{2}, 25\sqrt{2} \rangle$$

$$\boxed{\vec{R} = \langle 125\sqrt{3} - 25\sqrt{2}, 125 + 25\sqrt{2} \rangle}$$

$$\vec{R} = \langle 181.15, 160.36 \rangle$$

a) Magnitude $= |\vec{R}| = \sqrt{(181.15)^2 + (160.36)^2}$

$= 241.93$ Km/hr

$=$ speed of plane

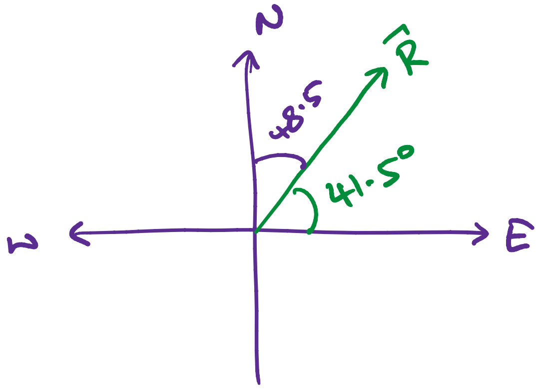
= speed of plane

b) Direction :

$$\theta = \arctan\left(\frac{160.36}{181.15}\right)$$

$$= 41.5^\circ$$

→ angle made with (+)ve x-axis



Bearings:

N $\underline{48.5^\circ}$ E

Ans.