Week-in-Review 2 (Vectors: part 2 and 3)

Problem 1. Find the dot product for the following pair of vectors. Are the vectors parallel, perpendicular or neither? If neither, find the angle between the vectors.

(1) $< 6, 0 >$ and $< 5, 3 >$

(2) $< 2, -1 >$ and $< -4, 2 >$

(3) $< 6, 2 >$ and $< 1, 3 >$

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Problem 2. What value(s) of x will make the vectors $< x, x >$ and $< 1, x >$ orthogonal?

Problem 3. Find the unit vector(s) orthogonal to the vector $< 3, 1 >$.

Find a unit vector parallel to the vector $< 3, 1 >$.

Problem 4. Find the scalar projection of $< 3, -1 >$ onto $< 2, 3 >$.

Find the vector projection of $< 2, 3 >$ onto $< 3, -1 >$.

Problem 5. A wagon is pulled a distance of 100 meters along a horizontal path by a constant force of 50 N. If the handle of the wagon is at an angle of 30° above the horizontal, how much work is done?

Problem 6. Given that the points $P(0,1), Q(2,1)$ and $R(5,4)$ make the 3 vertices of a triangle, find $\angle PQR$.

Problem 7. Find the vector equation of the line $2y + 3x = 5$

Problem 8. Find the vector equation of the line that makes an angle of $60°$ with the positive $x-$axis and passes through the point $(2, -5)$.

Problem 9. Find the Vector Equation of a line that passes through the point $(1, 3)$ and is parallel to the vector $< 1, -2 >$.

Find the Parametric Equations for this line.

Eliminate the parameter to find the Cartesian Equation of the line.

Problem 10. Find the distance of the point $(-2, 3)$ from the line $3x - 4y + 5 = 0$.

Problem 11. The position of an object moving in the $xy-$plane, after t seconds, is given by $\vec{r}(t) = (t+4)\vec{i} + (t^2+2)\vec{j}$.

(1) Find the position of the object at time $t = 2$.

(2) At what time will the object reach the point $(7, 11)$?

(3) At what time will the object reach the point $(9, 12)$?

(4) Find the Cartesian equation describing the path of the object.

Problem 12. Are the following pairs of lines parallel, perpendicular or neither? If the lines are not parallel, find the point of intersection between them.
$\vec{r_1}(t) = (-4 + 2t)\vec{i} + (5 + t)\vec{j}$ and $\vec{r_2}(s) = (2 + 3s)\vec{i} + (4 - 6s)\vec{j}$.