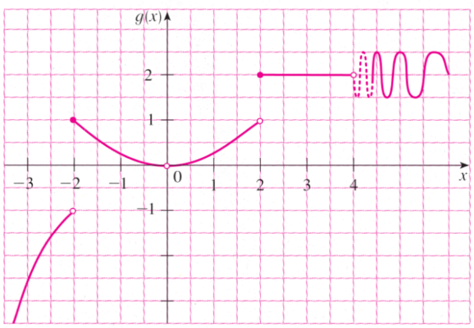
Week-in-Review 3
(Limits (2.2), Limit Laws (2.3) and Continuity (2.5))

Problem 1. For the function $g(x)$ whose graph is given below, state the value of the given quantity, if it exists.



a) $\lim_{x \to -2} g(x)$.

b) $g(-2)$

c) $\lim_{x \to 2} g(x)$.

d) $g(2)$.

e) $\lim_{x \to 0} g(x), g(0)$.

f) $\lim_{x \to 4^-} g(x)$, $\lim_{x \to 4^+} g(x)$.

Problem 2. Find the infinite limit for the following:

(a) $\lim_{x\to 0^-} \dfrac{x-1}{x^2(x+2)}$.

(b) $\lim_{x\to 0^+} \dfrac{x-1}{x^2(x+2)}$.

(c) $\lim_{x\to 0^-} \dfrac{x-1}{x(x+2)}$.

(d) $\lim_{x\to 0^-} \dfrac{x-1}{x(x+2)}$.

Problem 3. Given that $\lim_{x\to a} f(x) = -3$, $\lim_{x\to a} g(x) = 0$ and $\lim_{x\to a} h(x) = 8$, find the limits that exist.

(a) $\lim_{x\to a}[f(x) + h(x)]$

(b) $\lim_{x\to a}[f(x)]^2$

(c) $\lim_{x\to a} \sqrt[3]{h(x)}$

(d) $\lim_{x\to a} \dfrac{1}{f(x)}$

(e) $\lim_{x\to a} \dfrac{f(x)}{h(x)}$

(f) $\lim_{x\to a} \dfrac{g(x)}{f(x)}$

(g) $\lim_{x\to a} \dfrac{f(x)}{g(x)}$

(h) $\lim_{x\to a} \dfrac{2f(x)}{h(x) - f(x)}$

Problem 4. Evaluate the flowing limts if they exist:

(a) $\lim_{x \to 0} \dfrac{(2+x)^3 - 8}{x}$

(b) $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 3x + 2}$

(c) $\lim_{x \to 1} \left[\dfrac{1}{x-1} - \dfrac{2}{x^2 - 1}\right]$

(d) $\lim_{x \to 1.5} \frac{2x^2 - 3x}{|2x - 3|}$

(e) $\lim_{x \to 2^-} \sqrt{x^2 + x - 2}$

Problem 5. What is wrong with the following equation?

$$\frac{x^2 + x - 6}{x - 2} = x + 3$$

Problem 6. Use the Squeeze theorem to show that $\lim_{x \to 0} \sqrt{x^3 + x^2} \sin(\frac{\pi}{x}) = 0$

Problem 7. Given the piecewise function $h(x)$ below, evaluate the sunsequent limits, if they exist.

$$h(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \leq 2 \\ 8 - x & \text{if } x > 2 \end{cases}$$

(a) $\lim_{x \to 0^+} h(x)$.

(b) $\lim_{x \to 0} h(x)$.

(c) $\lim_{x \to 1} h(x)$.

(d) $\lim_{x \to 2^-} h(x)$.

(e) $\lim_{x \to 2^+} h(x)$.

(f) $\lim_{x \to 2} h(x)$.

Problem 8. Find the values of c and d that make the function $f(x)$ continuous everywhere.

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ cx^2 + d & \text{if } 1 \leq x \leq 2 \\ 4x & \text{if } x > 2 \end{cases}$$

Problem 9. Which of the following functions $f(x)$ has a removable discontinuity at $x = a$?

(a) $f(x) = \dfrac{x^2 - 2x - 8}{x + 2}$, $a = -2$

(b) $f(x) = \dfrac{x - 7}{|x - 7|}$, $a = 7$

(c) $f(x) = \dfrac{x^3 + 64}{x + 4}$, $a = -4$

(d) $f(x) = \dfrac{3 - \sqrt{x}}{9 - x}$, $a = 9$

Problem 10. If $f(x) = x^3 - x^2 - 10$, show that a number c exists such that $f(c) = 10$.

Problem 11. At what values of x is the following piece-wise function $f(x)$ discontinuous?

$$f(x) = \begin{cases} 5 & \text{if } \ x \leq -1 \\ -3x & \text{if } \ -1 < x \leq 1 \\ \dfrac{2x}{x-5} & \text{if } \ x > 1 \end{cases}$$

Problem 12. Test the following functions for continuity. If the function is discontinuous for some value $x = a$, explain why.

(a) $f(x) = \dfrac{x^2 - 1}{x + 1}, \quad a = -1$

(b) $f(x) = \begin{cases} 3 & \text{if } x = 4 \\ \dfrac{x^2 - 2x - 8}{x - 4} & \text{if } x \neq 4 \end{cases}, \quad \text{at } a = 4$