



WEEK-IN-REVIEW 4  
(LIMITS AT INFINITY (2.6), RATES OF CHANGE (2.7, 2.8) )

**Problem 1.** Find the following limits, if they exists.

(a)  $\lim_{x \rightarrow \infty} \frac{4x^2 + 3x + 5}{2 - x - 5x^2}$  .

(b)  $\lim_{x \rightarrow 5} \frac{2x^2 - 13x + 15}{x^2 - 3x - 10}$  .

(c)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + 3x + 1}}{7x - 3}$  .

2

$$(d) \lim_{x \rightarrow 2^+} \frac{2x}{4 - x^2} .$$

$$(e) \lim_{x \rightarrow \infty} \sqrt{x^2 + 3x + 1} - x .$$

$$(f) \lim_{x \rightarrow 0^-} \left( \frac{1}{x} - \frac{1}{|x|} \right) .$$

$$(g) \lim_{h \rightarrow 0} \frac{(5+h)^2 - 25}{h} .$$

$$(h) \lim_{x \rightarrow 7} \frac{\frac{1}{7} - \frac{1}{x}}{2x - 14} .$$

$$(i) \lim_{x \rightarrow 3} \frac{2x^2 - 6x}{|x - 3|} .$$

4

$$(j) \lim_{x \rightarrow \infty} \frac{1}{5 + e^{-x}} .$$

$$(k) \lim_{x \rightarrow \infty} \frac{3e^{2x} + e^{-7x}}{4e^{2x} - 3e^{-7x}} .$$

$$(l) \lim_{x \rightarrow -\infty} \frac{3e^{2x} + e^{-7x}}{4e^{2x} - 3e^{-7x}} .$$

$$(m) \lim_{x \rightarrow -3^-} e^{x/(x+3)} .$$

$$(n) \lim_{x \rightarrow \infty} [\ln(3x^6 + 1) - \ln(x^6 + 5)] .$$

$$(o) \lim_{x \rightarrow \infty} [\ln(3x^4 + 1) - \ln(x^6 + 5)] .$$

$$(p) \lim_{x \rightarrow \infty} [\ln(2^{3x} + 2)] .$$

$$(q) \lim_{x \rightarrow \infty} \arctan \left( \frac{5x^2 + 1}{5x^2 + 3} \right) .$$

6

**Problem 2.** Given the function  $f(x) = \sqrt{2x + 3}$ ,

(a) Use the limit definition of the derivative to find  $f'(x)$ .

(b) Find the equation of the tangent line to the function  $f(x)$  at  $x = 2$ .

**Problem 3.** The position function of a moving particle is given by  $f(t) = 4t^2 - 3t$ , where  $t$  represents time in seconds.

(a) Find the average velocity of the particle from  $t = 1$  to  $t = 4$ .

(b) Find the instantaneous velocity of the particle at time  $t = 2$ .

**Problem 4.** Given that  $f(x) = \frac{2}{5x+1}$ , use the limit definition of the derivative to find  $f'(x)$ .

**Problem 5.** Given that the graph of a function  $f(x)$  passes through the point  $(-1, 4)$ , and that the equation of a line tangent to  $f(x)$  at this point is given by  $y = 5 - 3x$ , what is

$$\lim_{x \rightarrow (-1)} \frac{f(x) - 4}{x + 1} = ?$$