



WEEK-IN-REVIEW 5: EXAM 1 REVIEW

Problem 1. Find the angle between the vectors $\langle 3, 1 \rangle$ and $-2\vec{i} + 2\vec{j}$.

Problem 2. What value(s) of x will make the vectors $x\vec{i} + \vec{j}$ and $(4 + x)\vec{i} + 3\vec{j}$ orthogonal?

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Problem 3. Find a vector of length 5 in the direction of the vector $\langle -3, 2 \rangle$.

Problem 4. Find the parametric equations of a line passing through the point $(1, 3)$ and perpendicular to the line $y = -3x + 1$.

Problem 5. How much work is done by a force of 10 N in order to push a box 15 m up a ramp, given that the ramp is inclined at an angle of 45° to the horizon?

Problem 6. Two ropes are used to suspend a 100 Kg weight. One rope makes an angle of 30° with the horizon while the other makes an angle of 60° with the horizon. Find the magnitude of tension in each rope.

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Problem 7. Find the vector equation of a line that passes through the points $(2, 5)$ and $(4, 7)$.

Problem 8. Express $\tan(\arcsin(2x))$ in terms of x .

Problem 9. Find the vertical and horizontal asymptotes of the function $f(x)$. Where is $f(x)$ discontinuous? When is the discontinuity removable?

a $f(x) = \frac{x^2 + 6x + 5}{x^2 - 3x - 4}$.

b $f(x) = \frac{\sqrt{x^2 + 2}}{3x - 6}$

Problem 10. Find the following limits, if they exist.

(a) $\lim_{x \rightarrow \infty} \frac{4x^2 + 3x + 5}{7 - 5x^2}$.

(b) $\lim_{x \rightarrow 5} \frac{2x^2 - 10x}{|5 - x|}$.

(c) $\lim_{x \rightarrow -\infty} \frac{7x - 3}{\sqrt{4x^2 + 3x + 1}}$.

$$(d) \lim_{x \rightarrow \infty} \arctan(e^x) .$$

$$(e) \lim_{x \rightarrow -\infty} \frac{3e^{-2x} + e^{7x}}{5e^{-2x} - 3e^{7x}} .$$

$$(f) \lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{x^2}\right) + 5 .$$

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(g) $\lim_{x \rightarrow \infty} [\ln(x^3 + 6) - \ln(2x^3 - 1)]$

(h) $\lim_{x \rightarrow \infty} \ln(5^x - 3)$

Problem 11. Given that $(3x + 2) \leq f(x) \leq (x^3 + 4)$ for $x \geq -2$, find $\lim_{x \rightarrow 1} f(x)$.

Problem 12. For what value(s) of x is $f(x)$ not continuous?

$$f(x) = \begin{cases} x + 2 & \text{if } x \leq -1 \\ \frac{|x - 1|}{x - 1} & \text{if } -1 < x \leq 1 \\ 0 & \text{if } x = 1 \\ -x^2 & \text{if } 1 \leq x < 2 \\ -2x - 3 & \text{if } x \geq 3 \end{cases}$$

Problem 13. Use the Intermediate Value Theorem to find an interval which contains the point of intersection of the functions $y = x^3 - 3x^2$ and $y = x - 5$.

Problem 14. Find the values of a and b that would make $f(x)$ continuous everywhere.

$$f(x) = \begin{cases} x + 3a & \text{if } x \leq 2 \\ ax^2 + bx + 2 & \text{if } 2 < x < 4 \\ 2bx - 2 & \text{if } x \geq 4 \end{cases}$$

Problem 15. Use the definition of the derivative to find $f'(x)$ for the function $f(x) = \sqrt{7+x}$,