151WIRH10

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151WIRH10

MATH 151- WEEK-IN-REVIEW 11

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EXAM 3 REVIEW

1. Approximate the area under the curve $f(x) = x^3 - 4$ on the interval $[-2, 6]$ using 4 equal-width rectangles and using left endpoints.

5.1/5.2

(a) 76
(b) 552
(c) 288
(d) 104
(e) 96

Δx

$\frac{b-a}{n} = \frac{6-(-2)}{4}$

$= 2$

Endpoints

$\boxed{-2}$ 0 2 4 $\cancel{6}$

Heights

$f(-2) = -12$
$f(0) = -4$
$f(2) = 4$
$f(4) = 60$

$2(-12) + 2(-4) + 2(4) + 2(60)$

Area $= 2(-12 - 4 + 4 + 60) = 96$

2. Find the intervals where $f(x)$ is increasing if $f'(x) = \frac{(x-4)^3(x+6)^8}{(7-x)^1}$. $= 0$ DNE

4.3

(a) $(-\infty, -6) \cup (7, \infty)$
(b) $(-6, 4) \cup (7, \infty)$
(c) $(4, 7)$
(d) $(-\infty, -6)$
(e) $(-\infty, 4) \cup (7, \infty)$

CN: $x = 4, -6, 7$

-7 $\boxed{-6}$ 0 4 5 7 8

$-$ $+$

$+$

3. Evaluate $\lim_{x\to\infty} x \sin\left(\frac{5}{x}\right)$

4.4

(a) 0
(b) ∞
(c) -5
(d) 5
(e) $-\infty$

$= \lim_{x\to\infty} \frac{\sin(\frac{5}{x})}{1/x}$

LH

$= \lim_{x\to\infty} \frac{\cos(\frac{5}{x})(+\frac{5}{\cancel{x^2}})}{+\frac{1}{\cancel{x^2}}}$

$= \lim_{x\to\infty} \left(\cos\left(\frac{5}{x}\right) \cdot 5\right)$

$\boxed{\frac{0}{0} \quad \frac{\infty}{\infty}}$ $\boxed{\infty^0}$ $\boxed{\infty - \infty}$ common denom $\boxed{1^\infty}$ ln

$\boxed{\infty \cdot 0}$ $\boxed{0^0}$

flip one over & put in denominator

4. Use the graph of $f(x)$ given to evaluate $\int_{-3}^{5} f(x)dx$.

$\frac{1}{2}(b_1+b_2)\cdot h$

5.2

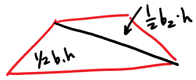
(a) $\frac{23}{2}$

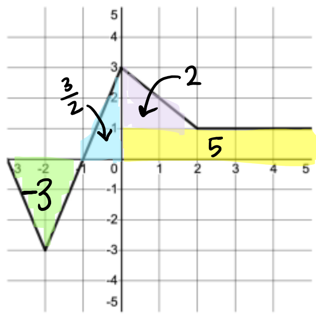
(b) $\frac{11}{2}$

(c) $\frac{17}{2}$

(d) $\frac{27}{2}$

(e) $\frac{15}{2}$





5. Find the value(s) of c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \sqrt{2x+3}$ on the interval $\left[\frac{1}{2}, 2\right]$.

$(2x)^{1/2}$

4.2

(a) 1

(b) $\frac{2}{9}$

(c) 0

(d) $\frac{9}{8}$

(e) The Mean Value Theorem is not valid for this function on the given interval.

$\frac{f(b)-f(a)}{b-a} = f'(c)$ $\left[\frac{1}{2}, 2\right]$

$\frac{5-4}{2-(\frac{1}{2})} = \boxed{\frac{1}{3/2}} = \frac{2}{3}$

$f'(x) = \frac{1}{\cancel{2}}(2x)^{-1/2}(\cancel{2}) = \frac{1}{(2x)^{1/2}}$

$\frac{1}{(2x)^{1/2}} = \frac{2}{3} \Rightarrow 3 = 2\sqrt{2x}$

$\frac{3}{2}\sqrt{2x}$

$\frac{9}{4} = 2x$

$x = \frac{9}{8}$

6. Find $f(0)$ if $f'(x) = \frac{1}{x^2+1} - 5 - \sin(x)$ and $f(1) = \cos(1)$

4.9

(a) $6 - \frac{\pi}{4}$

(b) $\frac{\pi}{4}$

(c) $4 + \frac{\pi}{2}$

(d) $5 - \frac{\pi}{2}$

(e) $\frac{\pi}{4} - 5$

$f(x) = \arctan(x) - 5x + \cos(x) + C$ $\quad 5 - \frac{\pi}{4}$

$f(1) = \arctan(1) - 5 + \cos(1) + C = \cos(1)$

$\frac{\pi}{4} - 5 + \cos(1) + C = \cos(1)$

$C = 5 - \frac{\pi}{4}$

$f(0) = \cancel{\arctan(0)} - \cancel{0} + \overset{1}{\cos(0)} + 5 - \frac{\pi}{4}$

7. Which of the following gives the exact area under the curve $f(x) = (x^3+5)^2$ on the interval $[1, 6]$?

5.1

(a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{6}{n}\left(\left(6+\frac{6i}{n}\right)^3+5\right)^2$

(b) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{5}{n}\left(\left(1+\frac{5i}{n}\right)^3+5\right)^2$

(c) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{6i}{n}\left(\left(1-\frac{6i}{n}\right)^3+5\right)^2$

(d) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{7}{n}\left(\left(6+\frac{7i}{n}\right)^3+5\right)^2$

(e) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{5i}{n}\left(\left(1+\frac{5}{n}\right)^3+5\right)^2$

Δx

$\frac{b-a}{n} = \frac{6-1}{n} = \frac{5}{n}$

Endpoints

$a + \Delta x \cdot i$

$= \left(1 + \frac{5i}{n}\right)$

Height

$\left(\left(1+\frac{5i}{n}\right)^3+5\right)^2$

$$\boxed{\text{Actual Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n}\left(\left(1+\frac{5i}{n}\right)^3+5\right)^2}$$

8. Given the following graph, determine where the function would have points of inflection given:

4.3

(a) the graph is $f(x)$

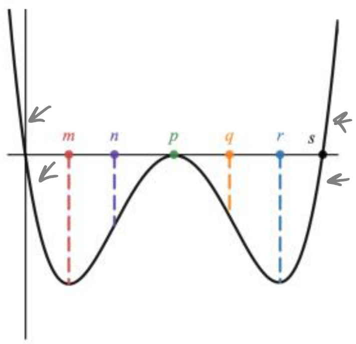
n , q

(b) the graph is $f'(x)$

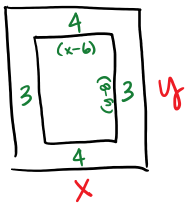
m p r

(c) the graph is $f''(x)$

O , s



9. The top and bottom margins of a poster are 4cm and the side margins are each 3cm. If the area of the printed material on the poster is fixed at 1200 cm^2, find the dimensions of the poster that will minimize the area of the full poster. Label the variables that you use in the picture. Clearly show or explain why your answer is a minimum.



OPTIMIZE

$A = x \cdot y$

$A = x\left(\frac{1200}{x-6} + 8\right)$

$= \frac{1200x}{x-6} + 8x$

$A' = \frac{(x-6)1200 - 1200x}{(x-6)^2} + 8$

$= \boxed{\frac{-7200}{(x-6)^2} + 8}$

CONSTRAINT

$(x-6)(y-8) = 1200$

$y - 8 = \frac{1200}{x-6}$

$y = \frac{1200}{x-6} + 8$

$0 = \frac{-7200}{(x-6)^2} + 8$

$-8 = \frac{-7200}{(x-6)^2}$

$(x-6)^2 = 900$

$x - 6 = \pm 30$

$\boxed{x = 36 \text{ cm}}, \cancel{-24}$

$y = \frac{1200}{30} + 8 = \boxed{48\text{ cm}}$

$-7200(x-6)^{-2} + 8$

(looking for ccu for minimum)

Second derivative test

$A'' = \frac{14400}{(x-6)^3}$

$A''(36) = \frac{14400}{30^3} > 0$

$\Rightarrow$ minimum

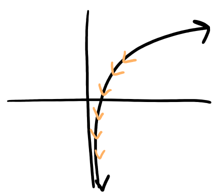
10. Evaluate the following limits.

(a) $\lim_{x \to \infty} [\ln(2x^2 + 9) - 3\ln(x + 1)]$

$= \lim_{x\to\infty} \left(\ln\left(\frac{2x^2+9}{(x+1)^3}\right)\right)$

$= \lim_{x \to 0^+} \ln(x)$

$= -\infty$



(b) $\lim\limits_{x\to 0^+} (1+\sin(x))^{3/x^2}$ $= 1^{\infty}$ (indeterminate)

$A =$

$\frac{0}{0}$ $\frac{\infty}{\infty}$

$\infty - \infty$

$\cancel{0^{\infty}}$ 0^0

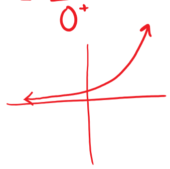
∞^0

$0 \cdot \infty$

$$\ln(A) = \lim_{x\to 0^+}\left(\frac{3}{x^2}\ln(1+\sin x)\right)$$

$$= \lim_{x\to 0^+}\left(\frac{3\ln(1+\sin(x))}{x^2}\right)$$

$$\overset{L'H}{=} \lim_{x\to 0^+}\left(\frac{\frac{3\cos(x)}{1+\sin(x)}}{2x}\right) = \frac{3}{0^+}$$



$$\ln(A) = \infty$$

$$A = e^{\infty} = \boxed{\infty}$$

(c) $\lim\limits_{x\to 0^+} (2^x - 1)\csc(x)$

11. Find $f(x)$.

(a) $f'(x) = 4^x + \sqrt[3]{x^4} - \frac{1}{7x} - \sin(x)$

$x^{4/3}$ $\frac{1}{7} \cdot \frac{1}{x}$

$f(x) = 4^x$ $f'(x) = 4^x \ln(4)$

$\frac{f'(x)}{\ln(4)} = 4^x$

$$\boxed{f(x) = \frac{4^x}{\ln(4)} + \frac{x^{7/3}}{7/3} - \frac{1}{7}\ln|x| + \cos(x) + C}$$

(b) $f'(x) = \frac{2x^3 - 7}{x^4} = \frac{2x^3}{x^4} - \frac{7}{x^4} = \frac{2}{x} - 7x^{-4}$

$$\boxed{f(x) = 2\ln|x| - \frac{7x^{-3}}{-3} + C}$$

(c) $f'(x) = x^2(8x - 3)$ and $f(1) = -1$

$f'(x) = 8x^3 - 3x^2$

$f(x) = 2x^4 - x^3 + C$

$2 - 1 + C = -1$

$C = -2$

$$\boxed{f(x) = 2x^4 - x^3 - 2}$$

(d) $f'(x) = 4x^3 + \frac{6}{1+x^2} - 7$ and $f(0) = 9$

$f(x) = x^4 + 6\arctan(x) - 7x + C$

$\frac{\sin}{\cos} = \frac{0}{1}$

$x = 0$

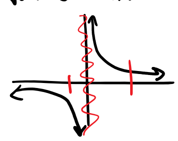
$f(0) = 0 + 0 - 0 + C = 9$

$C = 9$

$$\boxed{f(x) = x^4 + 6\arctan(x) - 7x + 9}$$

12. Find the value of c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \frac{4}{x}$ on the interval $[-1, 4]$.

Not continuous!



(a) 3

(b) -2

(c) 1

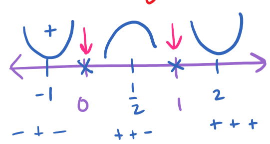
(d) 0

(e) The Mean Value Theorem is not valid for this function on the given interval.

13. The domain of $f(x)$ is all real numbers and $f''(x) = x^3(x^2+16)(x-1)$. Give the x-coordinate(s) of the inflection point.

$x^2+16=0$
$x^2=-16$

$x=0$ Never $=0$ $x=1$



(a) $x = 0$

(b) $x = 1$

(c) $x = -4, 0, 1, 4$

(d) $x = 0$ and $x = 1$

(e) $f(x)$ has no inflection points

Change + to − or − to +

14. Find the absolute extrema for $f(x) = \frac{4}{x} + \frac{x}{4} + 2$ on the interval $[1, 5]$.

$f = 4x^{-1} + \frac{1}{4}x + 2$

(a) The absolute maximum value is $\frac{25}{4}$ and the absolute minimum value is 4

(b) The absolute maximum value is $\frac{81}{20}$ and the absolute minimum value is 4

(c) The absolute maximum value is $\frac{25}{4}$ and the absolute minimum value is $\frac{81}{20}$

(d) The absolute maximum value is 6 and the absolute minimum value is $\frac{81}{20}$

(e) None of these.

$f' = -\frac{4}{x^2} + \frac{1}{4}$

$0 = -\frac{4}{x^2} + \frac{1}{4}$

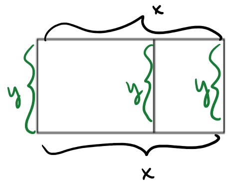
$x^2 = 16 \Rightarrow x = \pm 4$

$f(1) = 4 + \frac{1}{4} + 2 = \boxed{6\frac{1}{4}}$ Largest

$f(5) = \frac{4}{5} + \frac{5}{4} + 2 = \frac{41}{20} + \frac{40}{20} = \frac{81}{20}$

$f(4) = 1 + 1 + 2 = 4$ Smallest

15. A farmer has 4800 feet of fencing with which to enclose a rectangular field and divide the field as shown below. What dimensions of the field will maximize its area?



OPTIMIZE

$A = x \cdot y$

$A = x\left(1600 - \frac{2}{3}x\right)$

$A = 1600x - \frac{2}{3}x^2$

$\boxed{A' = 1600 - \frac{4}{3}x}$

$0 = 1600 - \frac{4}{3}x$

$x = 1200 \text{ ft}$

$-\frac{1600}{2(2/3)}$

CONSTRAINT

$4800 = 2x + 3y$

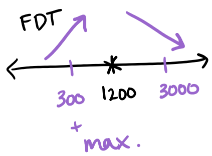
$4800 - 2x = 3y$

$y = 1600 - \frac{2}{3}x$

$y = 1600 - \frac{2}{3}(1200)$

$= 800 \text{ ft}$

FDT



16. Consider the function $f(x) = \frac{4}{x} + \frac{x}{4} + 2$.

(a) What is the domain of f?

D: $(-\infty, 0) \cup (0, \infty)$

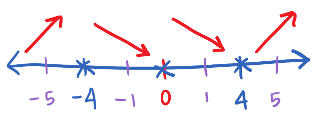
(b) Determine the interval(s) where f is increasing and decreasing and any local extrema.

$\boxed{f' = -4x^{-2} + \frac{1}{4}} \rightarrow f' = -\frac{4}{x^2} + \frac{1}{4} = \frac{-16 + x^2}{4x^2}$

$0 = -\frac{4}{x^2} + \frac{1}{4}$

$x^2 = 16$

$x = \pm 4$



Inc: $(-\infty, -4) \cup (4, \infty)$

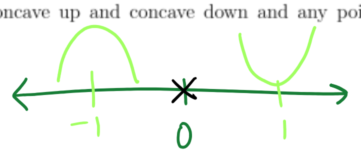
Dec: $(-4, 0) \cup (0, 4)$

Max @ $x = -4$

Min @ $x = 4$

(c) Determine the intervals where f is concave up and concave down and any points of inflection.

$f'' = \boxed{\frac{8}{x^3}}$



Not a POI
(not in domain)

CCU: $(0, \infty)$ ccd: $(-\infty, 0)$