Week 1

Monday, January 23, 2023 5:56 PM

151WIR_H1

TEXAS A&M UNIVERSITY
Math Learning Center

Math 151- Spring 2023
Week-In-Review 1

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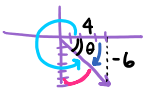
Problem Statements

1. Find each vector $\overrightarrow{AB}$
   (a) $A(-3,4)$, $B(1,-2)$

$\langle 1-(-3), -2-4 \rangle$

$\overrightarrow{AB} = \langle 4, -6 \rangle$



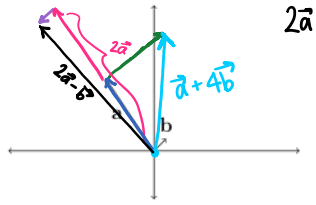
$\tan(\theta) = \frac{-6}{4}$

$\theta = \arctan\left(\frac{-6}{4}\right)$

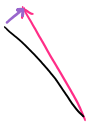
   (b) $A(-2,2)$, $B(-1,3)$

$\overrightarrow{AB} = \langle 1, 1 \rangle$ $\quad \|\overrightarrow{AB}\| = \sqrt{1^2 + 1^2} = \sqrt{2}$

2. Refer to the given drawing to find $2\mathbf{a} - \mathbf{b}$ and $\mathbf{a} + 4\mathbf{b}$.



$2\vec{a} - \vec{b} = 2\vec{a} + (-\vec{b})$



3. Given $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + 2\mathbf{j}$:

x-direction y-direction

   (a) Find $\|\mathbf{a}\|$.

$\|\vec{a}\| = \sqrt{3^2 + (-5)^2}$

$= \sqrt{34}$

   (b) Find a unit vector in the direction of $\mathbf{a}$.

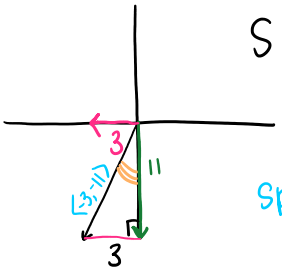
$\vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{1}{\|\vec{a}\|} \cdot \vec{a} = \frac{3\vec{i} - 5\vec{j}}{\sqrt{34}} = \frac{3}{\sqrt{34}}\vec{i} - \frac{5}{\sqrt{34}}\vec{j}$

   (c) Find $2\mathbf{a} + 4\mathbf{b}$.

$2(3\vec{i} - 5\vec{j}) + 4(-\vec{i} + 2\vec{j}) = 6\vec{i} - 10\vec{j} + (-4\vec{i}) + 8\vec{j}$

$= 2\vec{i} - 2\vec{j}$

4. Liam walks due west on the deck of a ship at 3 mph. The ship is moving south at 11 mph. Find the speed and direction of Liam relative to the surface of the water.



S ☐° W

arctan $\left(\frac{3}{11}\right)$

$\tan\theta = \frac{OPP}{ADJ} = \frac{-3}{-11} = \frac{3}{11}$

Speed = magnitude $= \sqrt{(-3)^2 + (-11)^2}$

$= \sqrt{9+121} = \sqrt{130}$ mph

5. Suppose a wind is blowing from the direction N60°W at a speed of 35 km/hr. A pilot is steering a plane in the direction S45°E at an airspeed (speed in still air) of 450 km/hr. Find the true course (the direction of the resultant of the velocity vectors of the plane and the wind) and the ground speed (the magnitude of the resultant) of the plane.

$\vec{w} = \langle 35\cos(150°), 35\sin(150°)\rangle$

$= \langle -35\cos(30°), +35\sin(30°)\rangle = \langle -35\cdot\frac{\sqrt{3}}{2}, 35\cdot\frac{1}{2}\rangle$

$\vec{p} = \langle 450\cos(45°), -450\sin(45°)\rangle$

$= \langle 450\cos(315°), 450\sin(315°)\rangle = \langle 450\cdot\frac{\sqrt{2}}{2}, -450\cdot\frac{\sqrt{2}}{2}\rangle$

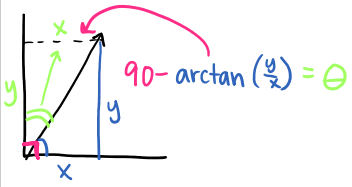
$= \langle 225\sqrt{2}, -225\sqrt{2}\rangle$

$\vec{t} = \langle -35\frac{\sqrt{3}}{2} + 225\sqrt{2}, 35\cdot\frac{1}{2} - 225\sqrt{2}\rangle$

ground speed = Velocity = $\sqrt{(\quad)^2 + (\quad)^2}$

true course: S arctan $\left(\frac{-35\frac{\sqrt{3}}{2}+225\sqrt{2}}{35\cdot\frac{1}{2}-225\sqrt{2}}\right)$° E

OR $90° - \arctan\left(\frac{y}{x}\right)$



6. Find $a \bullet b$ for the following vectors:

(a) $\mathbf{a} = \langle -1, 5\rangle$, $\mathbf{b} = \langle 6, 2\rangle$

$\vec{a}\cdot\vec{b} = a_1\cdot b_1 + a_2 b_2$

$= -1\cdot 6 + 5\cdot 2 = \boxed{4}$

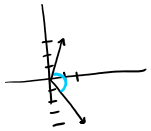
(b) $||a|| = 4$, $||b|| = 5$, and the angle between $\mathbf{a}$ and $\mathbf{b}$ is 30°

$\vec{a}\cdot\vec{b} = \|\vec{a}\|\cdot\|\vec{b}\|\cdot\cos\theta$

$= 4\cdot 5\cdot\cos(30°) = 20\cdot\frac{\sqrt{3}}{2} = \boxed{10\sqrt{3}}$

7. Find the angle between vectors:

(a) $\mathbf{a} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j}$



$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos\theta$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \dfrac{1 \cdot 2 + 3(-4)}{\sqrt{10}\ \sqrt{20}} = \dfrac{-10}{\sqrt{200}} = \dfrac{-10}{10\sqrt{2}} = \dfrac{-1}{\sqrt{2}} = \dfrac{-\sqrt{2}}{2}$

$\|\vec{a}\| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$\|\vec{b}\| = \sqrt{2^2 + (-4)^2} = \sqrt{20}$

$\cos\theta = \dfrac{-1}{\sqrt{2}}$

$\theta = \arccos\left(-\frac{1}{\sqrt{2}}\right) = 135°$

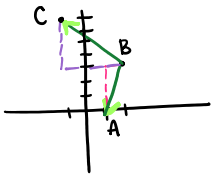
(b) $\mathbf{a} = \langle 2, 0\rangle$ and $\mathbf{b} = \langle 4, 1\rangle$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \dfrac{2 \cdot 4 + 0 \cdot 1}{\sqrt{4}\ \sqrt{17}}$

$= \dfrac{8}{2\sqrt{17}} = \dfrac{4}{\sqrt{17}}$

$\theta = \arccos\left(\frac{4}{\sqrt{17}}\right)$

8. Given the points $A(1, 0)$, $B(2, 3)$ and $C(-1, 7)$, find the angle $\angle ABC$.



$\dfrac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \cdot \|\vec{BC}\|} = \dfrac{-9}{\sqrt{10} \cdot 5} = \dfrac{-9}{5\sqrt{10}}$

$\vec{BA} = \langle -1, -3\rangle$

$\vec{BC} = \langle -3, 4\rangle$

$\theta = \arccos\left(\frac{-9}{5\sqrt{10}}\right)$

9. Find the value(s) of x that make(s) the vectors $\mathbf{a} = \langle 2x, -1\rangle$ and $\mathbf{b} = \langle x, 3x - 1\rangle$ orthogonal.

perpendicular

$\vec{a} \cdot \vec{b} = 0$

$(2x)(x) + (-1)(3x-1) = 0$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

$x = \frac{1}{2}, 1$

10. A force $\mathbf{F} = \langle -1, 2 \rangle$ is used to move an object from the point $(3, 0)$ to the point $(-2, 1)$. How much work is done by the force if distance is in meters and force is in Newtons?

$\text{Work} = \vec{F} \cdot \vec{D}$

$f_1 d_1 + f_2 d_2$ $\qquad$ $\|F\| \|D\| \cos\theta$

$\vec{F} = \langle -1, 2 \rangle \qquad \vec{D} = \langle -5, 1 \rangle$

$\text{Work} = (-1)(-5) + 2(1) =$ **7 Joules (Nm)**