Week 3

151WIRH3

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Math 151 - Week-In-Review 3

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Problem Statements

1. Find the limit.

(a) $\lim_{x\to 5} \frac{-1}{(x-5)^6}$

$\lim_{x\to 5^+} \frac{-1}{(x-5)^6}$

Try plugging in 5.1

$\frac{-1}{(5.1-5)^6} = \frac{-1}{(0.1)^6} = \frac{-1}{\text{tiny positive number}}$

$\lim_{x\to 5^+} \frac{-1}{(x-5)^6} = -\infty$

$\lim_{x\to 5^-} \frac{-1}{(x-5)^6}$

Try plugging in 4.9

$\frac{-1}{(4.9-5)^6} = \frac{-1}{(-0.1)^6} = \frac{-1}{\text{tiny positive number}}$

$\lim_{x\to 5^-} \frac{-1}{(x+5)^6} = -\infty$

$\boxed{\lim_{x\to 5} \frac{-1}{(x-5)^6} = -\infty}$

(b) $\lim_{x\to 5} \frac{-1}{(x-5)^5}$

$\lim_{x\to 5^+} \frac{-1}{(x-5)^5} = \frac{-1}{\text{tiny positive number}} = -\infty$

$\lim_{x\to 5^-} \frac{-1}{(x-5)^5}$

Try x= 4.9

$\frac{-1}{(-0.1)^5} = \frac{-1}{\text{tiny negative number}} = \infty$

$\boxed{\lim_{x\to 5} \frac{-1}{(x-5)^5} \text{ DNE}}$

$\lim_{x\to 5} \frac{x-6}{(x-5)^5}$

$= \lim_{x\to 5} \frac{-1}{(x-5)^5}$

(c) $\lim\limits_{x \to 5} \dfrac{x-5}{x^2+2x+1} = \dfrac{0}{36} = \boxed{0}$

(d) $\lim\limits_{t \to 0} \dfrac{t^2-5t}{t^2+3t} = \lim\limits_{t \to 0} \dfrac{\cancel{t}(t-5)}{\cancel{t}(t+3)} = \lim\limits_{t \to 0} \dfrac{t-5}{t+3} = \boxed{\dfrac{-5}{3}}$

New Variation $\lim\limits_{t \to 0} \dfrac{t(t-5)}{t^2(t+3)} = \lim\limits_{t \to 0} \dfrac{t-5}{t(t+3)} = \lim\limits_{t \to 0} \dfrac{-5}{t(3)}$ $\left.\begin{array}{l}\text{From left} \to \infty \\ \text{From right} \to -\infty\end{array}\right\} \Rightarrow \text{DNE}$

(e) $\lim\limits_{t \to 1} \dfrac{\sqrt{2-t}-1}{t-1}$

$= \lim\limits_{t \to 1} \dfrac{\sqrt{2-t}-1}{t-1} \cdot \dfrac{(\sqrt{2-t}+1)}{(\sqrt{2-t}+1)} = \lim\limits_{t \to 1} \dfrac{(2-t)-1}{(t-1)(\sqrt{2-t}+1)} = \lim\limits_{t \to 1} \dfrac{\overset{-(t-1)}{\cancel{1-t}}}{\cancel{(t-1)}(\sqrt{2-t}+1)} = \lim\limits_{t \to 1} \dfrac{-1}{\sqrt{2-t}+1} = \boxed{\dfrac{-1}{2}}$

(f) $\lim\limits_{h \to 0} \dfrac{(5-h)^{-1}-5^{-1}}{h}$

$\dfrac{1}{\cancel{5-h}}(\cancel{5-h})(5) = 5$

$= \lim\limits_{h \to 0} \dfrac{\left(\dfrac{1}{5-h} - \dfrac{1}{5}\right)(5-h)(5)}{(h)(5-h)(5)} = \lim\limits_{h \to 0} \dfrac{5-(5-h)}{h(5-h)(5)} = \lim\limits_{h \to 0} \dfrac{\cancel{h}}{\cancel{h}(5-h)(5)} = \lim\limits_{h \to 0} \dfrac{1}{(5-h)(5)} = \boxed{\dfrac{1}{25}}$

2. Find the following limits:

(a) $\lim\limits_{x \to 2}(3x^3 - 5x + 4)$

$= \boxed{18}$

(b) $\lim\limits_{x \to 1} \dfrac{x^2+x-2}{x+1} = \dfrac{0}{2} = 0$

$5 - t - 5$

(c) $\lim\limits_{t \to 0} \dfrac{\sqrt{5-t}-\sqrt{5}}{t} \cdot \dfrac{(\sqrt{5-t}+\sqrt{5})}{(\sqrt{5-t}+\sqrt{5})} = \lim\limits_{t \to 0} \dfrac{(5-t)-(5)}{t(\sqrt{5-t}+\sqrt{5})} = \lim\limits_{t \to 0} \dfrac{-\cancel{t}}{\cancel{t}(\sqrt{5-t}+\sqrt{5})}$

$= \lim\limits_{t \to 0} \dfrac{-1}{(\sqrt{5-t}+\sqrt{5})} = \boxed{\dfrac{-1}{\sqrt{5}+\sqrt{5}} = \dfrac{-1}{2\sqrt{5}}}$

(d) $\lim\limits_{x\to 3^+} \frac{|3-x|}{x-3}$

$|3-x| = |-(x-3)|$

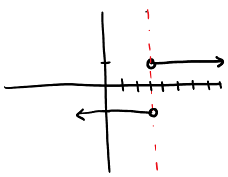
$$|3-x| = \begin{cases} 3-x & \text{if } x \le 3 \\ -(3-x) & \text{if } \underbrace{x > 3}_{3^+} \end{cases} \Rightarrow \lim_{x\to 3^+} \frac{|3-x|}{x-3} = \lim_{x\to 3^+} \frac{-(3-x)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{-3+x}{x-3}$$

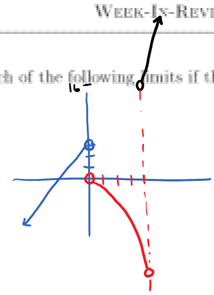
$$= \lim_{x\to 3^+} \frac{\cancel{x-3}}{\cancel{x-3}} = \boxed{1}$$

(e) $\lim\limits_{x\to 3^-} \frac{|3-x|}{x-3} = \lim\limits_{x\to 3^-} \frac{3-x}{x-3} = \lim\limits_{x\to 3^-} \frac{-\cancel{(x-3)}}{\cancel{x-3}} = \boxed{-1}$

(f) $\lim\limits_{x\to 3} \frac{|3-x|}{x-3}$ DNE



3. Let $f(x) = \begin{cases} x+3 & \text{if } x < 0 \\ -x^2 & \text{if } 0 < x < 4 \\ 4x & \text{if } x > 4 \end{cases}$ and evaluate each of the following limits if they exist.



(a) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x+3)$

$= 3$

(b) $\boxed{\lim_{x\to 0^+} f(x)} = \lim_{x\to 0^+} (-x^2)$

$= 0$

(c) $\lim_{x\to 0} f(x)$ DNE

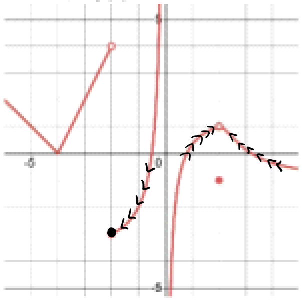
(d) $\lim_{x\to 2} f(x) = \lim_{x\to 2} (-x^2) = -4$

(e) $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (-x^2) = -4^2 = -16$

(f) $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (4x) = 4\cdot 4 = 16$

(g) $\lim_{x\to 4} f(x)$ DNE

4. Refer to the graph of $f(x)$ below. Find all values of x where $f(x)$ is discontinuous. For these values of x, is $f(x)$ continuous from the right, left or neither? Support your answer.



$x = -2$ Right continuous

Jump discontinuity

$x = 0$ Neither L nor R continuous

Vertical asymptote

$x = 2$ Neither L nor R continuous

(removable discontinuity)

Continuity Rules: ① $f(a)$ exists ② $\lim_{x \to a} f(x)$ exists ③ $f(a) = \lim_{x \to a} f(x)$

5. Determine whether the following functions are continuous at the indicated value of x. Support your answer.

(a) $f(x) = \begin{cases} \arctan(x) + 1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^3 & \text{if } x > 0 \end{cases}$ at $x = 0$

$f(x)$ is not continuous @ $x = 0$.

① $f(0) = 0$ ✓

② $\lim_{x \to 0} f(x)$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (\arctan(x) + 1) = \boxed{1}$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (1 - x^3) = \boxed{1}$

$\lim_{x \to 0} f(x) = 1$ ✓

③ ✗

① ≠ ②

(b) $f(x) = \dfrac{1}{x-1}$ at $x = 1$

$f(x)$ is not continuous @ $x = 1$

because f is undefined @ $x = 1$.

(c) $f(x) = \dfrac{x+4}{x^2+5x+4}$ at $x=-1$ and $x=-4$

$f(-1) = \dfrac{3}{0}$ Undefined $\Rightarrow$ f(x) is not continuous @ x=-1 because f(-1) DNE.

$f(-4) = \dfrac{0}{0}$ Undefined $\Rightarrow$ f(x) is not continuous @ x=-4 because f(-4) DNE

$\lim\limits_{x\to-4} \dfrac{\cancel{x+4}}{(x+1)\cancel{(x+4)}} = \lim\limits_{x\to-4} \dfrac{1}{x+1} = \dfrac{1}{-3}$

6. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-1}{x+1} & \text{if } x < -1 \\ ax^2+bx-5 & \text{if } -1 \le x \le 2 \\ 3x-a+2b & \text{if } x > 2 \end{cases}$$

Limits @ x= -1:

$\lim\limits_{x\to-1^-} \left(\dfrac{x^2-1}{x+1}\right) = \lim\limits_{x\to-1^-} \dfrac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}$

$= \lim\limits_{x\to-1^-} (x-1) = \boxed{-2}$

NEED $\lim\limits_{x\to-1^+} f(x) = -2$

$\lim\limits_{x\to-1^+} (ax^2+bx-5) = a-b-5$

$a-b-5=-2$

$\dfrac{11}{5}-b-5=-2$

$\dfrac{11}{5}-b=3$

$b=\dfrac{11}{5}-3 \quad \dfrac{15}{5}$

$\boxed{b=-\dfrac{4}{5}}$

Limits @ x= 2

$\lim\limits_{x\to2^-} (ax^2+bx-5)$

$= 4a+2b-5$

NEED $\lim\limits_{x\to2^+} f(x) = 4a+2b-5$

$\lim\limits_{x\to2^+} (3x-a+2b)$

$= 6-a+2b$

$4a+2b-5 = 6-a+2b$

$5a-11=0$

$5a=11$

$\boxed{a=\dfrac{11}{5}}$

$f(x) = \begin{cases} \dfrac{x^2-1}{x+1} & x \ne -1 \\ C & \boxed{x=-1} \end{cases}$

Hole@ (-1, -2)

y-value

$\lim\limits_{x\to-1} f(x) = f(-1)$

$C=-2$

$\dfrac{x^2-1}{x+1} \rightarrow \boxed{x-1}$