Math 151- Week-In-Review 5

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Derivatives

Basic Derivative Rules

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \ln(a)$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_b(x) = \frac{1}{x\ln(b)}$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

Inverse Trig Derivatives

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\csc^{-1}(x) = \frac{-1}{x\sqrt{x^2-1}}$$

Extra Derivative Rules

Product Rule

$$\frac{d}{dx}(f(x)\cdot g(x)) = f(x)\cdot g'(x) + f'(x)\cdot g(x)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

Chain Rule

$$\frac{d}{dx}(f(g(x))) = f'(g(x))\cdot g'(x)$$

1. Differentiate the following functions. You don't need to simplify.

(a) $f(x) = x^7 + \sqrt[4]{x} - \dfrac{5}{x} + \tan(x) - \csc(x)$

(b) $g(t) = (2t + 5)(3 - t)$

(c) $G(x) = \left(\dfrac{1}{x^2} - \dfrac{7}{x^5}\right)(3x - 2)$

(d) $h(y) = \dfrac{7}{y^4}$

(e) $y = \dfrac{\sqrt[4]{x^3} + x}{x^2}$

(f) $F(y) = \dfrac{x^2 - x}{4x + 5}$

(g) $q(t) = (2 - 3x + 5x^2)^{50}$

(h) $H(t) = \dfrac{x}{(x^3 - 7)^5}$

(i) $P(x) = e^{\tan(x)} - 3^{5x^2 - 1}$

(j) $f(x) = \ln(3x^2 + 5x - 1) + \log_4(3 - x)$

(k) $y = e^{t \sin^2(t)}$

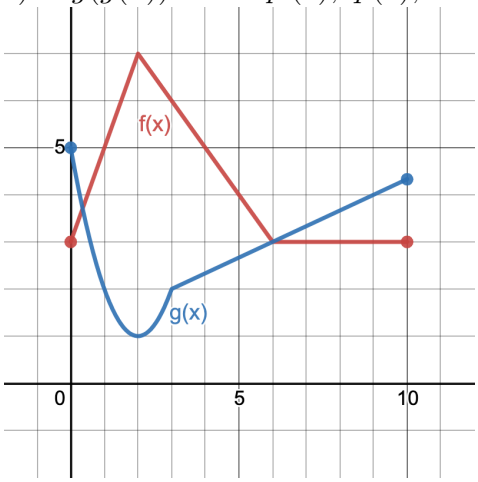
(l) $r(x) = \arccos(7x + 2)$

(m) $g(y) = \csc(\tan(\cos(y)))$

2. Find the 2023rd derivative of $y = 5\sin(5x)$

3. Find the 2023rd derivative of $y = xe^{2x}$

4. If f and g are the functions whose graphs are shown, let $p(x) = f(g(x))$, $q(x) = g(f(x))$, and $r(x) = g(g(x))$. Find $p'(2)$, $q'(9)$, and $r'(8)$.



5. Find $\dfrac{dy}{dx}$ for each of the following.

   (a) $5x^2 - x^3y + 5y = 7$

   (b) $\sec(xy) = 3 - \cos(y)$

6. Find the equation of the tangent line to $y = 3x + \sqrt{x}$ at $(4, 14)$.

7. Find the equation of the tangent line to $x^2 + 6y^2 = 25$ at $(1, 2)$.

8. Given $f(x) = \begin{cases} mx - b & \text{if } x < -1 \\ 5x^2 & \text{if } x \geq -1 \end{cases}$. Find values for m and b that make the function differentiable everywhere.

9. For what values of a and b is the line $y = 3x - 1$ tangent to the parabola $y = ax^2 + b$ when $x = 4$?

10. Show there are two tangent lines to the parabola $y = 4x^2$ that pass through the point $(0, -4)$. Find the equation of these tangent lines.