Math 151- Week-In-Review 7

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Exam 2 Review

1. Find the point(s) on the curve $x = t^2 + 4t$, $y = t^2 + 5t$ where the tangent line is vertical or horizontal.

2. Find the tangent vector of unit length for $\vec{r}(t) = \langle e^{t^2}, 3t\cos(t)\rangle$ at $t = 0$.
   (a) $\langle 0, 1\rangle$
   (b) $\langle 0, -3\rangle$
   (c) $\left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$
   (d) $\langle 0, -1\rangle$
   (e) $\left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right\rangle$

3. The radius of a sphere was measured to be 10in with a possible error of 0.25in. Use differentials to estimate the maximum error in the calculated surface area and find the relative error.

4. If $H(x) = f(g(x^2 + 4x))$, find $H'(1)$ given that $f'(1) = 2, f'(5) = 0, g(5) = 1, g'(1) = 4$, and $g'(5) = 3$.

(a) 36

(b) 2

(c) 0

(d) -8

(e) -30

5. Two sides of a triangle have length 8ft and 4ft. The angle between them at a rate of $\dfrac{\pi}{8}$rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\dfrac{\pi}{3}$.

6. Find the equation of the tangent line to the curve $2x^3y - 5y^4 = 11$ at the point $(2, 1)$.

7. Using differentials or a linear approximation, approximate $\sqrt[3]{10}$.

8. A particle moves according to the equation $s(t) = t^2 - 4t + 1$ where t is measured in seconds and s is in feet. What is the total distance the particle travels in the first 3 seconds?

9. Calculate the 77^{th} derivative of $g(x) = 2\sin(4x)$.
   (a) $g^{(77)} = 2^{78}\cos(4x)$
   (b) $g^{(77)} = -2^{78}\sin(4x)$
   (c) $g^{(77)} = 2^{155}\cos(4x)$
   (d) $g^{(77)} = -2^{155}\cos(4x)$
   (e) $g^{(77)} = 2^{155}\sin(4x)$

10. At what point on the graph of $f(x) = \ln(x)$ is the tangent line parallel to the line $x + 5y = 3$?

11. A bacteria culture doubles every 6 hours. How long will it take to triple in size?

12. Use logarithmic differentiation to find the derivative of each of the following.
    (a) $y = (3x + 1)^{\tan(x)}$

(b) $y = (\ln(x))^{x^4-7}$

13. Find the quadratic with equation $y = ax^2+bx$ whose tangent line at $x = 2$ has equation $y = 4x+6$.

14. If $f(x) = \ln(\sin^2(x))$, find $f'(\frac{\pi}{2})$.
   (a) $f'(\frac{\pi}{2}) = 2$
   (b) $f'(\frac{\pi}{2}) = -1$
   (c) $f'(\frac{\pi}{2}) = \frac{1}{2}$
   (d) $f'(\frac{\pi}{2}) = 0$
   (e) $f'(\frac{\pi}{2}) = 2\sqrt{3}$

15. Find $\dfrac{dy}{dx}$ for $\sin(xy^3) - \tan(4x) = 2x^3 + 3^{y^2}$.

16. Find the equations of the lines through the point $(2, -7)$, that are tangent to the parabola $y = x^2 - x$.

17. Find the derivative of $y = \arccos(e^{3x})$.

(a) $f'(x) = \dfrac{3e^{3x}}{\sqrt{1+e^{6x}}}$

(b) $f'(x) = \dfrac{3e^{3x}}{\sqrt{1-e^{6x}}}$

(c) $f'(x) = -\dfrac{3e^{3x}}{\sqrt{1-e^{6x}}}$

(d) $f'(x) = -\dfrac{3}{\sqrt{1-e^{6x}}}$

(e) $f'(x) = -\dfrac{3e^{3x}}{1+e^{6x}}$

18. Find $f''(0)$ if $f(x) = (5 - x^2)^3$.

19. A camera is positioned 3000 feet from the base of a rocket launching pad. At a particular moment, the rocket rises vertically. Its speed is 1500 ft/s when it has risen 4000 ft.

    (a) How fast is the distance from the camera to the rocket changing at that moment?

    (b) If the camera is focused on the rocket, how fast is the camera's angle of elevation changing at that moment?

20. $f(x) = \begin{cases} ax^2 - 2bx + 8 & \text{if } x \leq 2 \\ bx - 1 & \text{if } x < 2 \end{cases}$

(a) What must be true for $f(x)$ to be continuous everywhere?

(b) Find the values of a and b that make $f(x)$ differentiable everywhere, if possible. If not possible, explain why.

21. Find the derivative of $f(x) = \ln\left(\dfrac{e^{3x}(2x+7)^4}{\sqrt[3]{x^2-5}}\right)$