MATH 151- WEEK-IN-REVIEW 8

ALEXANDRA L. FORAN

WHAT DOES THE DERIVATIVE TELL US?

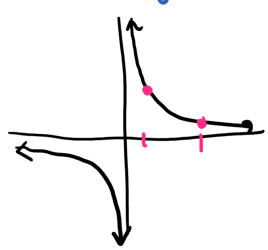
1. Find the absolute maximum and minimum values of each of the following functions on the given interval.

   (a) $y = \frac{1}{x}$ on $[1, 5]$

EVT : If y is continuous on $[a,b]$ then it will have both an abs. max & absolute minimum on $[a,b]$.

$y = x^{-1}$

$\hookrightarrow y' = -1x^{-2} = \frac{-1}{x^2}$



Critical #

$y' = 0$

$-\frac{1}{x^2} = 0$

$-1 \neq 0$

Never $= 0$

y' DNE

$x^2 = 0$

$x = 0$

Crit # @ $x = 0$

Check y-values:

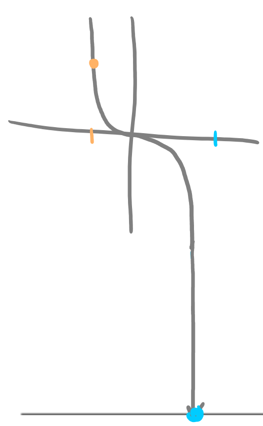
$y(1) = 1$ Abs. max value

$y(5) = \frac{1}{5}$ Abs. min value

   (b) $f(x) = -5x^3$ on $[-2, 4]$

$f'(x) = -15x^2$

c.v. @ $x = 0$

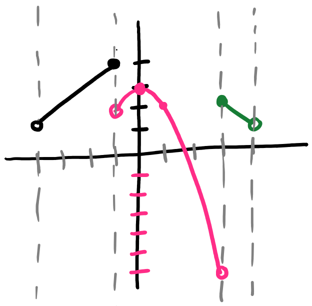


$f(0) = 0$

$f(-2) = \boxed{40}$ Abs. max value (Abs max @ $x = -2$)

$f(4) = \boxed{-320}$ Abs min value (Abs min @ $x = 4$)

(c) $g(x) = \begin{cases} x+5 & -4 < x \leq -1 \\ 3 - x^2 & -1 < x < 3 \\ 5 - x & 3 \leq x < 4 \end{cases}$



Abs max @ $x=-1$
Abs max value $= 4$

No absolute minimum.

(d) $h(x) = x^2 e^{-x}$ on $[0, 4]$

$h'(x) = x^2 \cdot e^{-x}(-1) + 2x e^{-x}$

$0 = \boxed{e^{-x} \cdot x \cdot (-x+2)}$

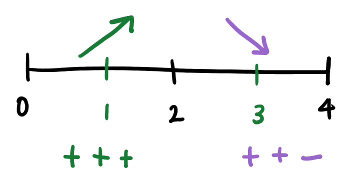
No crit values from e^{-x} $\qquad$ $x=0$ $\qquad$ $x=2$

$h'(x) = e^{-x} \cdot x \cdot (-x+2)$

$h(0) = 0$ Absolute minimum

$h(2) = 4e^{-2} = \frac{4}{e^2}$ ← Absolute maximum

$h(4) = 16e^{-4} = \frac{16}{e^4}$



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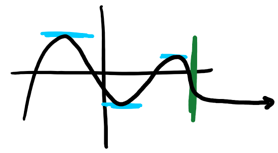
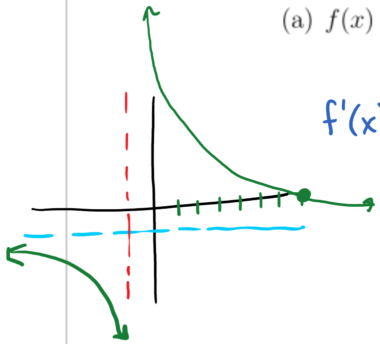


2. Find the critical values of the following functions.

(a) $f(x) = \dfrac{7-x}{x+1}$



$$f'(x) = \frac{(x+1)(-1) - (7-x)(1)}{(x+1)^2}$$
$$= \frac{-x-1-7+x}{(x+1)^2}$$
$$= \frac{-8}{(x+1)^2}$$

f' DNE @ $x=-1$
NOT a critical value because $x=-1$ is not in the domain!

No local max/min → $\boxed{\text{No critical values.}}$

(b) $g(x) = (x^3 - 12x)^{1/3}$

Domain: $(-\infty, \infty)$

$$g'(x) = \frac{1}{3}(x^3-12x)^{-2/3} \cdot (3x^2-12)$$
$$= \frac{x^2-4}{(x^3-12x)^{2/3}}$$

$g' = 0$
$x^2-4=0$
$x^2=4$
$\boxed{x=\pm 2}$

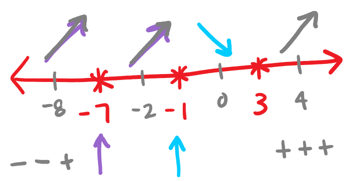
← C.V. →

g' DNE
$x^3-12x=0$
$x(x^2-12)=0$
$\boxed{x=0,\ x=\pm\sqrt{12}=\pm 2\sqrt{3}}$

3. Classify the local extrema of $f(x)$ given $f'(x) = (x-3)^5(x+1)(x+7)^8$

Domain: $(-\infty, \infty)$

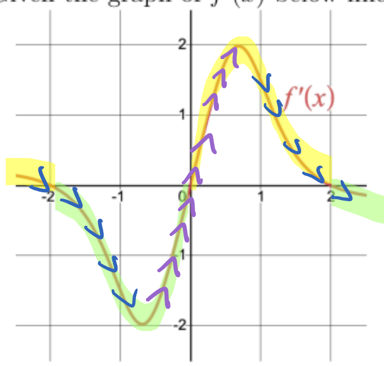
C.V. $x = 3, -1, -7$



Inc: $(-\infty,-7)\cup(-7,-1)\cup(3,\infty)$
Dec: $(-1,3)$

$\boxed{\begin{array}{l}\text{Rel/local max @ } x=-1 \\ \text{Rel/local min @ } x=3\end{array}}$

4. Given the graph of $f'(x)$ below find the given intervals/values.



f	f'
$\nearrow$	$+$
$\searrow$	$-$
$\cup$	$\nearrow$
$\cap$	$\searrow$

(a) Intervals where $f(x)$ is increasing

$(-\infty, -2) \cup (0, 2)$

(b) Intervals where $f(x)$ is decreasing

$(-2, 0) \cup (2, \infty)$

(c) x-values of any local maxima

$x = -2, 2$

(d) x-values of any local minima

$x = 0$

(e) Intervals where $f(x)$ is concave up

$(-\frac{3}{4}, \frac{3}{4})$

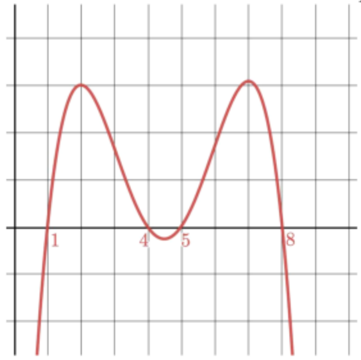
(f) Intervals where $f(x)$ is concave down

$(-\infty, -\frac{3}{4}) \cup (\frac{3}{4}, \infty)$

(g) x-values of any points of inflection

$x = -\frac{3}{4}, \frac{3}{4}$

5. Give the x-value of the inflection points of f for each part.



(a) The above curve is the graph of f.

$x = 3.25,\ 5.75$

$\frac{4.5+7}{2} = \frac{11.5}{2}$

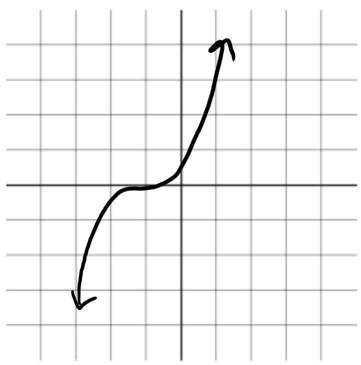
(b) The above curve is the graph of f'.

$x = 2,\ 4.5,\ 7$

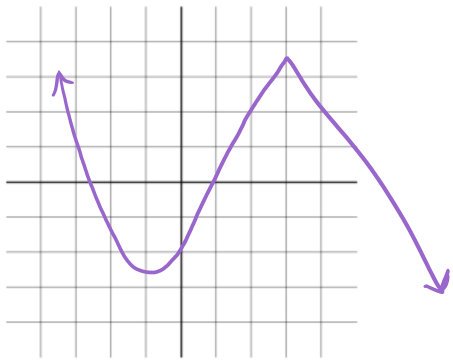
(c) The above curve is the graph of f''.

$x = 1,\ 4,\ 5,\ 8$

6. Sketch a graph of a continuous function where $x = -1$ is a critical number, but the function has no local extrema.



7. Sketch a graph of a continuous function where $x = -1$ is a local minimum and the function is not differentiable at $x = 3$.



8. If $f'(x) = x(4x-1)^{2/3}$, find where the function is concave up. Are there any points of inflection?

$f''(x)$

$$f''(x) = 1(4x-1)^{2/3} + x\cdot\frac{2}{3}(4x-1)^{-1/3}\cdot 4$$

$$= (4x-1)^{-1/3}\left[(4x-1)^{1} + \frac{8}{3}x\right]$$

$\frac{2}{3}-\left(-\frac{1}{3}\right)$

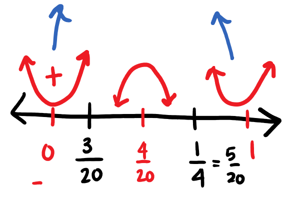
$$= \frac{4x-1+\frac{8}{3}x}{(4x-1)^{1/3}} = \boxed{\frac{\frac{20}{3}x-1}{(4x-1)^{1/3}}}$$

← $f''=0$ $\quad x=\frac{3}{20}$

← f'' DNE $\quad x=\frac{1}{4}$

BOTH POI

CCU: $\left(-\infty,\frac{3}{20}\right)\cup\left(\frac{1}{4},\infty\right)$



9. If $f(x) = x^2 \ln\left(\frac{x}{4}\right)$, find where the function is concave up. Are there any points of inflection?

$(0,\infty)$

$$f'(x) = x + \ln\left(\tfrac{x}{4}\right)(2x)$$

$$f''(x) = 1 + \ln\left(\tfrac{x}{4}\right)\cdot 2 + \frac{1}{x}\cdot 2x$$

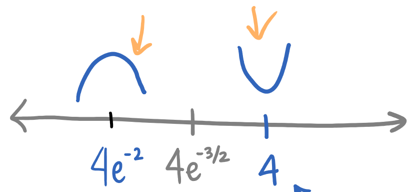
$$f''(x) = \boxed{3+2\ln\left(\tfrac{x}{4}\right)}$$

$$0 = 3+2\ln\left(\tfrac{x}{4}\right)$$

$$-\frac{3}{2} = \ln\left(\tfrac{x}{4}\right)$$

$$e^{-3/2} = \frac{x}{4} \Rightarrow x = 4e^{-3/2}$$

POI



$3+2\ln\left(\frac{\cancel{4}e^{-2}}{\cancel{4}}\right)$
$=3+2(-2)$
$=-1$

$3+2\ln(1)$
$=3$

CCU: $\left(4e^{-3/2},\infty\right)$

POI