Math 251 – Spring 2021 Week In Review

Wir 11: Chapter 16: 16.1-16.9

Problem 1. Evaluate $\int_C y ds$, where C is parameterized by $\mathbf{r}(t) = \langle t, t^3 \rangle$, $0 \le t \le 1$.

Problem 2. Find $\int_C x ds$, where C is the right half of the circle $x^2 + y^2 = 4$, oriented counterclockwise.

Problem 3. Evaluate $\int_C z dx + (xy) dy$, where C is the line segment from (-1,1,0) to (1,2,0).

Problem 4. Find $\int_C (3y + 7e^{\sqrt{x}})dx + (8x + 9\cos(y^2))dy$, where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$.

Problem 5. Evaluate $\int_C (xy) dx + (x-y)dy$, where C is the line segment from (1,1) to (2,0) and then from (2,0) to (3,5).

Problem 6. A particle starts at the point (-3,0), moves along the x-axis to the point (3,0), then along the semicircle $y = \sqrt{9-x^2}$, then back to the starting point. Find the work done on this particle by the force field $\mathbf{F} = \langle 3x, x^3 + 3xy^2 \rangle$.

Problem 7. Find the work done by the force field $\mathbf{F} = \langle x^2, y^2 \rangle$ in moving a particle along the arc of the parabola $y = 2x^2$ from the point (-2, 8) to (1, 2).

Problem 8. Given $\mathbf{F} = \langle 4xe^z, \cos(y), 2x^2e^z \rangle$ and $\mathbf{r}(t) = \langle \sin(t), t, \cos(t) \rangle$, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $0 \le t \le \frac{\pi}{2}$.

Problem 9. Find the surface area of the part of the plane 6x + 2y + 8z = 24 in the first octant.

Problem 10. Find the surface area of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

Problem 11. Set up but do not evaluate an integral which gives is the correct set up in order to evaluate $\iint_S yz \, dS$ where S is the part of the sphere $x^2 + y^2 + z^2 = 16$ that lies between the planes z = 2 and $z = 2\sqrt{3}$. Note: If we parameterize the sphere $x^2 + y^2 + z^2 = \rho^2$ by $\mathbf{r}(\theta, \phi) = \langle \rho \sin(\phi) \cos(\theta), \rho \sin(\phi) \sin(\theta), \rho \cos(\phi) \rangle$, then $|\mathbf{r}_{\theta} \times \mathbf{r}_{\phi}| = \rho^2 \sin(\phi)$.

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Problem 12. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle y, x, z \rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ between the planes z = 1 and z = 4.

Problem 13. Find the flux of $F = \langle x, y, -z \rangle$ across S, where S is the part of the paraboloid $z = 4 - x^2 - y^2$ that is above the xy-plane. Use the positive (outward) orientation.

Problem 14. Use Stokes' Theorem to set up but not evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle xz, 2xy, 3xy \rangle$ and where C is the boundary curve of the part of the plane 3x + y + z = 3 in the first octant. Note: Your limits of integration must be defined with the appropriate differential.

Problem 15. Set up but do not evaluate the integral which is the result of using Stokes' Theorem to find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle \ 2xz, \ 4x^2, \ 5y^2 \rangle$ and C is curve of intersection of the plane z = x+4 and the cylinder $x^2 + y^2 = 4$, oriented counterclockwise when viewed from above.

Problem 16. Use Stokes' Theorem evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle x^2 \sin(z-5), y^2, xy \rangle$ and S is the part of the paraboliod $z = 9 - x^2 - y^2$ that lies above the plane z = 5, oriented upward.

Problem 17. Using the The Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

 $\mathbf{F} = \left\langle 4x, \sin(e^z), \sqrt{x^3 + y^2} \right\rangle$ and S is the surface bounded by $x^2 + y^2 = 4$, z = 2, z = 4.

Problem 18. Using the The Divergence Theorem, find the flux of $\mathbf{F} = \langle ye^{z^2}, ze^x, 2z + 8 \rangle$ across S, where S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 9$, z = 0 and z = y - 4.